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Reaction Sphere Actuator ⋆

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Abstract: The reaction sphere actuator (RSA) described in the paper is a novel device for satellite attitude control. It consists of a sphere, an actuation system, made out of holonomic wheels, and an outer encapsulating shell. Reaction torques are obtained by adequate rotation of the sphere. The mechanical rolling contact between the actuation system and the sphere also ensures the sphere static equilibrium and the transfer of the reaction torque to the encapsulating shell. RSA reaction torques can be generated along any rotational axis. This paper starts by discussing the controllability of the system, which amounts to a feasibility of the concept, goes over the actuation system, basic control strategy and simulation results. Finally a performance analysis comparison between a RSA and an equivalent reaction wheel arrangement is presented.

Keywords: Satellite Dynamics, Attitude Control, Reaction Sphere, Satellite Actuator.

1. INTRODUCTION

Satellite attitude control systems usually consider reaction torque devices (RTDs) to adjust their attitude. Reaction wheel arrangements (RWAs) are nowadays commonly used. A single reaction wheel (RW) with rotational axis fixed, can induce attitude changes to a coupled body around that same rotational axis. Combining multiple RWs along independent axis, it is possible to achieve three-axis attitude motion capability. In general, redundant arrangements are used with more than 3 RWs. Optimal criteria can be used to select one solution among all RWs torque combinations, Landis Markley and Lebsock (2010). A geometry example of redundant RWAs is based on a regular tetrahedral geometry configuration, Kök (2012).

The reaction sphere actuator (RSA) actuation system has close ties with the Ballbots robots, Fankhauser and Gwerder (2010). While a Ballbot robot navigates by actuating its sphere, the RSA aims to move its sphere in order to produce reaction torques and apply them to coupled bodies. Although their purposes are distinct, there is a relation between both devices actuating systems since they both rely upon holonomic wheels (see Fankhauser and Gwerder (2010), Lauwers et al. (2006) for additional information on Ballbot robots with connections to the device presented in this paper).

Among the related work, the European Levitated Spherical Actuator (ELSA) project, comprising a magnetic spherical rotor that reacts to an induced electric-magnetic field generated from the involving stator, also aims to take advantage of a spherical reaction body. Although having similarities compared to the RSA in this paper, the ELSA project concept presents difficulties concerning sphere rotor control, due to its magnetic dynamic model Leopoldo Rossini and Perriard (2013).

This paper covers a RSA kinematic and dynamic analysis, showing both the device feasibility, from a geometrical viewpoint, and its advantages compared to other RTDs. The RSA consists of a sphere, an actuation system, and an outer shell that encapsulates both. The actuation system comprises four or more holonomic wheels in mechanical contact with the sphere surface, conferring three independent rotational degrees-of-freedom (DOF) to the sphere. Each holonomic wheel is actuated by a motor rigidly attached to the RSA outer shell. The coupling ensures the transfer of the sphere angular momentum to any body attached to the outer shell, e.g., the main body of a satellite. Furthermore, the number of actuators and their placement determines the sphere static equilibrium and its property to rotate without displacing its geometric center with respect to a frame fixed in the outer shell.

Section 2 illustrates a kinematic analysis proving the RSA feasibility. Section 3 addresses satellite attitude dynamics and corresponding attitude control scheme. Section 4 presents simulation results showing both the satellite attitude model validity and a RTDs performance comparison. Section 5 concludes the paper.

2. KINEMATIC MODEL

The kinematic analysis is divided in two stages, corresponding to two subsystems (i) a sphere rolling on a plane, and (ii) a mobile robot with a regular arrangement of holonomic (swedish) wheels, equivalent to the actuation system.

2.1 Rolling Contact Kinematics

The kinematic analysis of a rolling sphere on a plane requires the definition of two surfaces, namely a sphere

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surface and a plane. The relative motion between both surfaces is defined according to local coordinate frames defined in each surface at the point of contact. The evolution of each surface contact point coordinates can be written in terms of surface geometric parameters, (M_i, K_i, T_i) denoting respectively surface i metric tensor, curvature tensor and torsion tensor, see Carmo (1976) for surface parametrizations details. Two surfaces (S and P) in contact have kinematics equations, Montana (1988),

$$\begin{bmatrix}\dot{u}_{\mathrm{S}}\\ \dot{v}_{\mathrm{S}}\end{bmatrix} = M_{\mathrm{S}}^{-1}(K_{\mathrm{S}}+\tilde{K_{\mathrm{P}}})^{-1}\left(\begin{bmatrix}-w_y\\ w_x\end{bmatrix} - \tilde{K_{\mathrm{P}}}\begin{bmatrix}v_x\\ v_y\end{bmatrix}\right), \quad (1)$$

$$\begin{bmatrix}\dot{u}_{\mathrm{P}}\\ \dot{v}_{\mathrm{P}}\end{bmatrix} = M_{\mathrm{P}}^{-1}R_{\psi}(K_{\mathrm{S}}+\tilde{K_{\mathrm{P}}})^{-1}\left(\begin{bmatrix}-w_y\\ w_x\end{bmatrix} + K_{\mathrm{S}}\begin{bmatrix}v_x\\ v_y\end{bmatrix},\right) \quad (2)$$

$$\dot{\psi} = w_z + T_{\mathrm{S}}M_{\mathrm{S}}\begin{bmatrix}\dot{u}_{\mathrm{S}}\\ \dot{v}_{\mathrm{S}}\end{bmatrix} + T_{\mathrm{P}}M_{\mathrm{P}}\begin{bmatrix}\dot{u}_{\mathrm{P}}\\ \dot{v}_{\mathrm{P}}\end{bmatrix}, \quad (3)$$

$$v_z = 0, \quad (4)$$

$$\tilde{K_{\mathrm{P}}} = R_{\psi}K_{\mathrm{P}}R_{\psi}, \quad (5)$$

$$R_{\psi} = \begin{bmatrix}cos(\psi) & -sin(\psi)\\ -sin(\psi) & -cos(\psi)\end{bmatrix}, \quad (6)$$

where $(u_{\mathrm{S}}, v_{\mathrm{S}}, u_{\mathrm{P}}, v_{\mathrm{P}})$ denote respectively the contact point coordinates in the sphere and plane surface parametrizations. Velocities ω_x, ω_y, ω_z, v_x, v_y, v_z define respectively the angular and linear velocities between the sphere and plane local coordinate frames and ψ represents the angle between both surfaces local coordinate frames.

2.2 Controllability of the Rolling Sphere

This subsection illustrates the controllability analysis of a rolling sphere kinematic model using Lie Algebra concepts (see for instance Sussmann (1987) for a definition of controllability).

According to (1)-(6) the rolling sphere kinematic model yields,

$$\begin{bmatrix}\dot{u_{\mathrm{S}}}\\ \dot{v_{\mathrm{S}}}\\ \dot{u_{\mathrm{P}}}\\ \dot{v_{\mathrm{P}}}\\ \dot{\psi}\end{bmatrix} = -\begin{bmatrix}0 & r & 0\\ r & 0 & 0\\ \frac{\sin(\psi)}{\cos(v_{\mathrm{P}})} & \frac{\cos(\psi)}{\cos(v_{\mathrm{P}})} & 0\\ \cos(\psi) & -\sin(\psi) & 0\\ \sin(\psi)\tan(v_{\mathrm{P}}) & \cos(\psi)\tan(v_{\mathrm{P}}) & 1\end{bmatrix}\begin{bmatrix}w_x\\ w_y\\ w_z,\end{bmatrix} \quad (7)$$

where r denotes the sphere radius, and ω_x,ω_y, ω_z denote the sphere angular velocities projected upon the world frame, Svinin and Hosoe (2007).

The kinematic model (7) is a 3-input 5-output system, thus it is composed of three vector fields describing three independent motions, yielding

$$\dot{x} = g_1(x)\omega_x + g_2(x)\omega_y + g_3(x)\omega_z \quad (8)$$

where g_1, g_2 and g_3 denote the model vector fields (columns of matrix in (7)), and x represents the system variables.

Following *Chow* theorem, forming a full rank controllability matrix requires five independent vector fields, which can be computed as combinations of Lie bracket operators, Murray et al. (1994) Siciliano et al. (2009).

One example of a controllability matrix is given by the following combinations of Lie brackets of the vector fields in (7),

$$[[g_1]\ [g_2]\ [g_3]\ [g_1, g_2]\ [g_1, [g_1, g_2]]] \quad (9)$$

Hence the rolling sphere is controllable with three non-coplanar angular velocities (w_x,w_y and w_z), to be controlled by the actuation system. The two additional vector fields in the basis of the Lie Algebra represent complex maneuvering required to achieve the extra motion.

2.3 Controllability of the Actuation System

This subsection describes a kinematic model for an arrangement of generic holonomic wheels as commonly used in omnidirectional mobile robots, i.e. the roller axis angle and wheel geometrical arrangement are defined as variables (see Figure 1).

The rotational axis are fixed with respect to each other and lie always parallel to the fixed ground plane. Moreover each holonomic wheel is indexed from 1 to N (for details on this kinematic model applied to mobile robots see Indiveri (2009)).

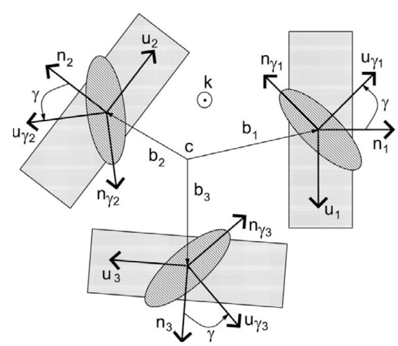

Fig. 1. Three-wheel omnidrive robot (Indiveri (2009))

The holonomic mobile robot kinematic model is,

$$M\begin{bmatrix}v_c\\ \omega\end{bmatrix} = \rho\,\dot{\mathsf{q}}\,\cos(\gamma) \quad (10)$$

where ρ corresponds to the wheel radius, $\dot{\mathsf{q}}$ denotes the wheel joint velocity (hub axis) and,

$$M = -\begin{bmatrix}n_{\gamma x1} & n_{\gamma y1} & b_1^T u_{\gamma 1}\\ n_{\gamma x2} & n_{\gamma y2} & b_2^T u_{\gamma 2}\\ \vdots & \vdots & \vdots\\ n_{\gamma xN} & n_{\gamma yN} & b_N^T u_{\gamma N}\end{bmatrix} \in \mathbb{R}^{N\times 3} \quad (11)$$

The model is controllable if, Indiveri (2009),

(1) $cos\gamma \neq 0$
(2) rank $M = 3$

An equilateral triangle wheel arrangement is a case of a controllable wheel geometry arrangement suitable for actuating the RSA sphere, see Campion et al. (1996).

2.4 RSA controllability

The actuation system is fixed relative to the RSA outer shell, with the actuators in contact with the surface of the