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Two-Level Nonlinear Tracking Control of a Quadrotor Unmanned Aerial Vehicle

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Abstract: This study presents a nonlinear tracking control design for a quadrotor unmanned aerial vehicle (UAV) with a two-layer control architecture. In order to facilitate the control design, a decoupled dynamic model is derived for the quadrotor UAV. Backstepping method is employed to stabilize the closed-loop system and to achieve output tracking. The control structure has a high-level layer for producing the guiding state trajectories, and a low-level layer for tracking in both altitude and attitude dynamics. The designed control scheme can well handle the model nonlinearities as well as the drag effects, and can be directly implemented to produce the PWM control signals. Simulation results are provided to verify the efficiency of the proposed control scheme.

Keywords: Quadrotor, Unmanned Aerial Vehicle, Backstepping Control.

1. INTRODUCTION

Quadrotor unmanned aerial vehicle (UAV) systems, because of being capable to take off and land vertically and hover at close proximity of specified locations in 3D as well as having favorable accurate dynamic models and stability characteristics, build compared to single rotor and fixed-wing UAVs. Therefore, quadrotors have been popular for researchers and customers in the last decade. Developments and performance requirements of quadrotors have been increased for difficult missions and environments in research studies and commercial use. Due to the simple modular and motion capabilities, quadrotors have been used in various complicated indoor and outdoor tasks.

In the literature, various methods from nonlinear control theory have been applied to motion control of quadrotors. In Madani et al. (2006), a full state backstepping control design has been applied to stabilize the system dynamics and to track arbitrary reference trajectories. In Bouabdallah et al (2007), an integral backstepping control is proposed for position control of an autonomous quadrotor. A feedback linearization-based controller is designed together with high order sliding mode observer (SMO) against external disturbances such as wind and noise in Benallegue et al (2008). Ryan et al (2013) presents a linear matrix inequality (LMI) based control design using approximate feedback linearization. Lee et al (2013) presents a global dynamic model for a quadrotor UAV, and it develops robust nonlinear tracking controllers to avoid singularities for complex maneuver motions. A nonlinear model based adaptive controller for attitude regulation is presented in Zeng et al (2011). As a nonlinear autonomous control design, Choi et al (2015) presents a backstepping-like feedback linearization method to stabilize the quadrotor. Choi et al (2015) also implements its control design in the fully autonomous real-time tests.

As we can see from the existing literature (Lee et al (2013); Zeng et al (2011); Raffo et al (2010); Besnard et al (2012); Choi et al (2015)), it is not an easy task to design a controller for the full motion dynamics of a quadrotor, including model nonlinearities and drag effects. It is also challenging to further implement the controller on a real system. In this study, the objective is to design a two level control scheme for tracking and stabilization of a quadrotor UAV considering model nonlinearities and external drag effects.

Compared with other related works (Köksal et al (2015); Güler et al (2013)), the two-level control structure simplifies the design process of the controller for a quadrotor. In this control scheme, the high-level control loop deals with the generation of the desired positions and attitude angles, while the low-level is responsible for the altitude and attitude dynamics of the quadrotor. To generate the control command that matches the pulse-width modulation (PWM) signals of the motor drivers, actuator dynamics are also considered at design level, which makes the designed controller more reliable. Both altitude and attitude controllers are designed using back-stepping method.

The main advantage of the proposed control scheme over the existing literature is decoupling the system dynamics into three sub-models. These sub-systems ease the control design and analysis for each sub-model separately.

The rest of the study is organized as follows: the modeling of the quadrotor and the control objective are presented in Section II. Design processes of high-level loop and low-level loop are given in Section III and Section IV, respectively.

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Simulation is implemented in Section V to illustrate the effectiveness of the designed controller. This study is finally concluded in Section VI.

2. DYNAMIC MODELLING AND CONTROL OBJECTIVES

2.1 Dynamic Model of the Quadrotor

The coordinates of the quadrotor system's body frame $\{O_b, x_b, y_b, z_b\}$ centered at the center of gravity (CG) of the quadrotor, the global frame $\{O_g, x, y, z\}$, thrusts, moments and gravity are represented in Fig. 1. Using Euler angles $\varphi \triangleq [\phi, \theta, \psi]^T$ and the rotational matrix $R \in SO(3)$ from the the body frame to the global frame, and following the Newton-Euler formalism, the dynamic model is derived based on applied forces $F \in \mathbb{R}^3$ and moment $M \in \mathbb{R}^3$ (Köksal et al (2015)), and is given by

$$F = RF_b = m\ddot{p} \;\; and \;\; M = J\dot{w} + w \times Jw \tag{1}$$

where R is the rotational matrix; $F_b = [F_{xb}, F_{yb}, F_{zb}]^T = [0, 0, \sum_{i=1}^{4} T_i]^T$ is the applied force vector generated by actuators' thrust forces $T_i,\ i = 1, 2, 3, 4$, in the body frame; m is the total mass of the system; $J_\varphi = diag(J_\phi, J_\theta, J_\psi)$ is the rotational inertia matrix in the body frame; $w = [\dot{\phi}, \dot{\theta}, \dot{\psi}]^T$ is the angular velocity of O_b. (1) leads to the following equations of motion (Koksal et al (2015)):

$$\begin{aligned}
\ddot{x} &= \frac{(T_1 + T_2 + T_3 + T_4)(\sin\psi\sin\phi + \cos\phi\sin\theta\cos\psi)}{m},\\
\ddot{y} &= \frac{(T_1 + T_2 + T_3 + T_4)(-\sin\phi\cos\psi + \cos\phi\sin\theta\sin\psi)}{m},\\
\ddot{z} &= \frac{(T_1 + T_2 + T_3 + T_4)(\cos\phi\cos\theta)}{m} - g,\\
\ddot{\phi} &= \frac{l(T_1 - T_2)}{J_\phi} + \frac{(J_\theta - J_\psi)\dot{\psi}\dot{\theta}}{J_\phi} - d_\phi\dot{\phi},\\
\ddot{\theta} &= \frac{l(T_3 - T_4)}{J_\theta} + \frac{(J_\psi - J_\phi)\dot{\psi}\dot{\phi}}{J_\theta} - d_\theta\dot{\theta},\\
\ddot{\psi} &= \frac{K_\psi(T_1 + T_2 - T_3 - T_4)}{J_\psi} + \frac{(J_\phi - J_\theta)\dot{\theta}\dot{\phi}}{J_\psi} - d_\psi\dot{\psi},
\end{aligned} \tag{2}$$

where $p = [x, y, z]^T$ is the position of O_b; $\varphi \triangleq [\phi, \theta, \psi]^T$ are the Euler angles of rotation; $\beta = [d_\phi, d_\theta, d_\psi]^T$ are rotational drag parameters; $T_i,\ i = 1, ., 4$ are thrust forces generated on each actuator; l is the distance between the center of gravity (O_b) and each propeller; K_ψ is thrust-to-moment gain; g is gravitational acceleration.

Assumption 1. *It is assumed that attitude angles are limited as*

$$-\frac{\pi}{2} < \phi < \frac{\pi}{2}, \quad -\frac{\pi}{2} < \theta < \frac{\pi}{2}, \; and \; -\pi < \psi \leqslant \pi. \tag{3}$$

Besides, as an attempt to generate thrust forces using actuators, we use the first-order thrust-input model (Quanser (2013)) in the Laplace domain as follows:

$$T_i(s) = K\frac{b}{s + b}v_i(s) \tag{4}$$

where b is the actuator bandwidth; K is a positive armature gain.

In the control design, we decouple the attitude and the altitude dynamics. Using the control inputs of decoupled dynamics, a PWM input generator is obtained as

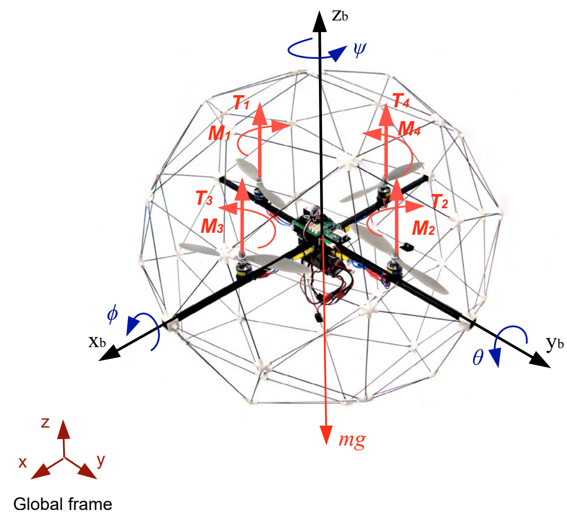

Fig. 1. The Qball-X4 quadrotor (Quanser (2013)).

$$v = Gu = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & -1 & 1 & 1 \\ 1 & 0 & -1 & 1 \\ -1 & 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} u_\varphi \\ u_z \end{bmatrix}, \tag{5}$$

where $v = [v_1, v_2, v_3, v_4]^T \in \mathbb{R}^4$ is PWM input for each actuator; G is the PWM generator matrix; $u_\varphi = [u_\phi, u_\theta, u_\psi]^T \in \mathbb{R}^3$ is attitude control inputs; $u_z \in \mathbb{R}$ is altitude control input. Employing (5) we map the generated control signals $u = [u_\varphi^T, u_z]^T$ to the actual PWM signals v for the four motors.

Similar to (5), we define the effective altitude thrust T_z and attitude thrusts $T_\varphi = [T_\phi, T_\theta, T_\psi]^T$ as follows:

$$T_z \triangleq (T_1 + T_2 + T_3 + T_4)/4, \tag{6}$$

$$T_\phi \triangleq (T_1 - T_2)/2, \tag{7}$$

$$T_\theta \triangleq (T_3 - T_4)/2, \tag{8}$$

$$T_\psi \triangleq (T_1 + T_2 - T_3 - T_4)/4. \tag{9}$$

Combining the nonlinear dynamics (2), thrust-input model (4) and the relation (6)-(9), we derive the nonlinear state variable model as

$$\dot{X} = F(X, u) = \begin{bmatrix} X_3 \\ X_4 \\ \frac{4}{m}f_1(X_2)X_6 - \zeta \\ Af_2(X_4) + BX_4 + \Gamma X_5 \\ -bX_5 + Kbu_\varphi \\ -bX_6 + Kbu_z \end{bmatrix}, \tag{10}$$

with state vector

$$X = [X_1, X_2, X_3, X_4, X_5, X_6]^T \in \mathbb{R}^{16}, \tag{11}$$

where $X_1 = p = [p_l^T, p_z]^T, X_2 = \varphi \triangleq [\phi, \theta, \psi]^T, X_3 = \dot{X}_1 = v, X_4 = \dot{X}_2 = w_\varphi, X_5 = T_\varphi \triangleq [T_\phi, T_\theta, T_\psi]^T$ are 3-D vectors; $X_6 = T_z$; $\zeta = [0, 0, g]^T$; $f_1(X_2) = \begin{bmatrix} \cos\phi\sin\theta\cos\psi + \sin\phi\sin\psi \\ \cos\phi\sin\theta\sin\psi - \sin\phi\cos\psi \\ \sin\phi\cos\theta \end{bmatrix}$; $f_2(X_4) = \begin{bmatrix} \dot{\theta}\dot{\psi} \\ \dot{\phi}\dot{\psi} \\ \dot{\phi}\dot{\theta} \end{bmatrix}$; $A = diag(\frac{J_\theta - J_\psi}{J_\phi}, \frac{J_\psi - J_\phi}{J_\theta}, \frac{J_\phi - J_\theta}{J_\psi})$; $B = diag(-d_\phi, -d_\theta, -d_\psi)$ $\Gamma = diag(\frac{2l}{J_\phi}, \frac{2l}{J_\theta}, \frac{4K_\psi}{J_\psi})$.