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Linear Modeling of a Flexible Substructure Actuated through Piezoelectric Components for Use in Integrated Control/Structure Design

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Abstract This study presents a generic TITOP (Two-Input Two-Output Port) model of a substructure actuated with embedded piezoelectric materials as actuators (PEAs), previously modeled with the FE technique. This allows intuitive assembly of actuated flexible substructures in large flexible multi-body systems. The modeling technique is applied to an illustrative example of a flexible beam with bonded piezoelectric strip and vibration attenuation of a chain of flexible beams.

Keywords: Finite Element, Piezoelectric actuators and sensors (PEAs), Multi-body Flexible Structures, Flexible Structures Control, TITOP modeling

1. INTRODUCTION

Piezoelectric actuators and sensors (PEAs) have been widely used in the field of system control design of large flexible structures. However, the design of control systems involving PEAs requires an accurate knowledge of the electro-mechanical behaviour of the system for vibration dynamics, transfers between the inputs and the outputs and non-linear effects such as hysteresis and creep effect. In order to integrate PEAs in the controlled structure, a design procedure including virtual prototyping of piezoelements integrated with the structure needs to be developed.

Macroscopic PEAs models are divided in two main categories. In the first category, the behaviour of a PEAs is decoupled in several contributions such as hysteresis, vibration dynamics and creep based on physical laws. The most well-known model structure of PEAs is the electromechanical model proposed in Goldfarb and Celanovic (1997), in which all effects are taken into account. In this category, other models only consider vibration dynamics with Finite Element (FE) models (Piefort and Preumont (2001)) or static behaviour (Smits et al. (1991)). The second category does not decouple the different behaviours of the PEA, all effects are considered simoultaneously. However, they are only accurate over small frequency ranges, what seriously limit their usage.

This study presents a PEAs modelisation technique that allows considering piezoelectric actuated flexible substructures linked with other substructures. Taking advantage of the TITOP modeling technique Perez et al. (2015b); Alazard et al. (2015); Perez et al. (2015a), the method casts in state-space form the FE model of an actuated flexible substructure in order to consider the acceleration-loads transfer within a flexible-multibody system. This supposes an extension of the TITOP modeling technique to piezoelectric materials, completing the available modeling techniques for flexible multibody control. Section 2 introduces the main equations of a piezoelectric FE model. Then, Section 3 explains how to obtain the Two-Input Two-Output Port(TITOP) model through Component Modes Synthesis (CMS) and the double-port approach. Then, an application for a beam with bonded piezoelectric strip is illustrated. Finally, application to vibration attenuation of a chain of flexible beams is performed and conclusions are stated.

2. FINITE ELEMENT MODELING OF A PIEZOELECTRIC COMPONENT

As stated in IEEE and Engineers (1988), the constitutive linear equations of an element piezoelectric material read:

$$\{T\} = \left[c^E\right]\{S\} - \left[e\right]^T\{E\} \tag{1}$$

$$\{D\} = \left[e\right]\{S\} + \left[\epsilon^S\right]\{E\} \tag{2}$$

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where $\{T\}$ is the stress vector, $\{S\}$ the deformation vector,$\{E\}$ the electric field, $\{D\}$ the electric displacement, $[c]$ the elasticity constants matrix, $[\epsilon]$ the dielectric constants matrix, $[e]$ the piezoelectric constants, with superscripts ^{E}, ^{S} and ^{T} indicating static conditions for E, S and T respectively.

The dynamic equations of a piezoelectric continuum can be discretized in elements and written in the finite element formulation as follows (Piefort and Preumont, 2001):

$$\left[\mathcal{M}_{qq}\right]\left\{\ddot{q}\right\}+\left[\mathcal{K}_{qq}\right]\left\{q\right\}+\left[\mathcal{K}_{q\phi}\right]\left\{\phi\right\}=\left\{f\right\} \tag{3}$$

$$\left[\mathcal{K}_{\phi q}\right]\left\{q\right\}+\left[\mathcal{K}_{\phi\phi}\right]\left\{\phi\right\}=\left\{\gamma\right\} \tag{4}$$

where the element coordinates $\{q\}$, the applied voltage $\{\phi\}$, the electric charge $\{\gamma\}$ and external forces $\{f\}$ are related through the element mass matrix, $[\mathcal{M}_{qq}]$, the element stiffness matrix, $[\mathcal{K}_{qq}]$, the piezoelectric coupling matrix $[\mathcal{K}_{q\phi}]$ and the capacitance matrix $[\mathcal{K}_{\phi\phi}]$. Upon carrying out the assembly of each piezoelectric element, we get the global system of equations:

$$\left[M_{uu}\right]\left\{\ddot{u}\right\}+\left[K_{uu}\right]\left\{u\right\}+\left[K_{uv}\right]\left\{v\right\}=\left\{F\right\} \tag{5}$$

$$\left[K_{vu}\right]\left\{u\right\}+\left[K_{vv}\right]\left\{v\right\}=\left\{g\right\} \tag{6}$$

where the global coordinates $\{u\}$, the global applied voltage $\{v\}$, the electric charge $\{g\}$ and external forces $\{F\}$ are now related through the global mass matrix, $[M_{uu}]$, the global stiffness matrix, $[K_{uu}]$, the piezoelectric coupling matrix $[K_{uv}]$ and the capacitance matrix $[K_{vv}]$.

3. TITOP MODELING OF THE PIEZOELECTRIC COMPONENT

The Two-Input Two-Output Port (TITOP) model is a linear modeling tool developed with the objective of providing fundamental bricks for the modeling and assembly of flexible multibody systems. As demonstrated in Perez et al. (2015b), the TITOP model is as accurate as other accepted methods (the assumed modes method), more robust to variations in boundary conditions and it is applicable in cases of small deformation and large overall motion, as in a two-link flexible arm or a satellite with flexible appendages. In this study, an extension to the case of piezoelectric components is proposed in order to complete the modeling tools. This is done through the application of the Component Modes Synthesis (CMS) transformation to Eqns. (5) and (6), then casting the resulting transformation into a state-space representation for the desired inputs-outputs.

3.1 Component Modes Synthesis Transformation

The Component Modes Synthesis transformation allows separating the different contributions of elastic body displacements into rigid body, redundant boundaries and internal elastic displacements. The resulting equations are easier to manipulate since rigid-body and elastic displacements appear uncoupled in the transformed stiffness matrix. The fundamentals of Component Modes Synthesis were stated by Hurty (1965) in 1965 and then recalled later by Craig Jr (2000), the reader might consult those references if more information about CMS is desired.

The global coordinates $\{u\}$ are then partitioned into three main sets: rigid-body coordinates, r, redundant boundary coordinates, c, and fixed-interface normal modes, n. Applying this division, Eqns. (5) and (6) result:

$$\begin{bmatrix} M_{nn} & M_{nc} & M_{nr} \\ M_{cn} & M_{cc} & M_{cr} \\ M_{rn} & M_{rc} & M_{rr} \end{bmatrix}_{uu} \begin{Bmatrix} \ddot{u}_n \\ \ddot{u}_c \\ \ddot{u}_r \end{Bmatrix} + \begin{bmatrix} K_{nn} & K_{nc} & K_{nr} \\ K_{cn} & K_{cc} & K_{cr} \\ K_{rn} & K_{rc} & K_{rr} \end{bmatrix}_{uu} \begin{Bmatrix} u_n \\ u_c \\ u_r \end{Bmatrix} + \begin{bmatrix} K_{nv} \\ K_{cv} \\ K_{rv} \end{bmatrix} \left\{v\right\} = \begin{Bmatrix} F_n \\ F_c + \tilde{F}_c \\ F_r + \tilde{F}_r \end{Bmatrix} \tag{7}$$

$$\begin{bmatrix} K_{vn} & K_{vc} & K_{vr} \end{bmatrix} \begin{Bmatrix} u_n \\ u_c \\ u_r \end{Bmatrix} + \left[K_{vv}\right]\left\{v\right\} = \left\{Q\right\} \tag{8}$$

where $[M]$, $[K]$, $\{u\}$ and $\{F\}$ have been partitioned into their contributions to rigid-body, redundant boundaries and fixed-boundary displacements. The "tilde" load term, $\tilde{F}_r$ and $\tilde{F}_c$, denotes the force resulting from the connection to adjacent structures at the boundary points.

In CMS, physical displacements can be expressed in terms of generalized coordinates by the Rayleigh-Ritz coordinate transformation Craig Jr (2000):

$$\begin{Bmatrix} u_n \\ u_c \\ u_r \end{Bmatrix} = [\Phi] \begin{Bmatrix} \eta_n \\ \eta_c \\ \eta_r \end{Bmatrix} = \begin{bmatrix} \phi_{nn} & \phi_{nc} & \phi_{nr} \\ 0_{cn} & I_{cc} & \phi_{cr} \\ 0_{rn} & 0_{rc} & I_{rr} \end{bmatrix} \begin{Bmatrix} \eta_n \\ \eta_c \\ \eta_r \end{Bmatrix} \tag{9}$$

where the component-mode matrix $[\Phi]$ is a matrix of preselected component modes including: fixed-constraint modes,n, redundant boundary modes, c, and rigid-body modes, r. Pre-multiplying by $[\Phi]^T$, substituting Eqn. (9) into Eqns. (7) and (8) and considering that neither interior forces nor external forces apply ($F_n = F_c = F_r = 0$), Eqn. (7) yields:

$$\begin{bmatrix} \hat{M}_{nn} & \hat{M}_{nc} & \hat{M}_{nr} \\ \hat{M}_{cn} & \hat{M}_{cc} & \hat{M}_{cr} \\ \hat{M}_{rn} & \hat{M}_{rc} & \hat{M}_{rr} \end{bmatrix}_{\eta\eta} \begin{Bmatrix} \ddot{\eta}_n \\ \ddot{\eta}_c \\ \ddot{\eta}_r \end{Bmatrix} + \begin{bmatrix} \hat{K}_{nn} & 0_{nc} & 0_{nr} \\ 0_{cn} & \hat{K}_{cc} & 0_{cr} \\ 0_{rn} & 0_{rc} & 0_{rr} \end{bmatrix}_{\eta\eta} \begin{Bmatrix} \eta_n \\ \eta_c \\ \eta_r \end{Bmatrix} + \begin{bmatrix} \hat{K}_{nv} \\ \hat{K}_{cv} \\ \hat{K}_{rv} \end{bmatrix} \left\{v\right\} = \begin{Bmatrix} 0 \\ \tilde{F}_c \\ \tilde{F}_r + \phi_{cr}^T \tilde{F}_c \end{Bmatrix} \tag{10}$$

$$\begin{bmatrix} \hat{K}_{nv} & \hat{K}_{cv} & \hat{K}_{rv} \end{bmatrix} \begin{Bmatrix} \eta_n \\ \eta_c \\ \eta_r \end{Bmatrix} + \left[K_{vv}\right]\left\{v\right\} = \left\{G\right\} \tag{11}$$

with the new coupling matrix coefficients:

$$\hat{K}_{nv} = \phi_{nn} K_{nv} \tag{12}$$

$$\hat{K}_{cv} = K_{cv} + \phi_{cn} K_{nv} \tag{13}$$

$$\hat{K}_{rv} = K_{rv} + \phi_{rc} K_{cv} + \phi_{rn} K_{nv} \tag{14}$$

Equations (10) and (11) are the most generalized expression for a FE model of a piezoelectric component transformed through the CMS method. Section 3.2 will show how to take advantage of this form in order to simply model an accurate piezoelectric component for control of flexible multi-body systems.