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A Box Regularized Particle Filter for terrain navigation with highly non-linear measurements ⋆

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Abstract: This paper addresses the design of a new set-membership particle filter named Box Regularized Particle Filter (BRPF) applied to terrain navigation. This algorithm combines the set-membership particle estimation (known as Box Particle Filter) with the Kernel estimation method. This approach makes possible to enhance significantly the filter's robustness while reducing the computation time (only 200 particles are needed instead of 5,000 with a conventional Sequential Importance Resampling (SIR) Particle Filter). Numerical results are presented from 10,000 Monte-Carlo runs.

Keywords: Particle Filter, Set-Membership Estimation, Regularized Particle Filter, Inertial Navigation Update, Radar Altimeter, Digital Terrain Elevation Data, Monte Carlo methods.

1. INTRODUCTION

A variety of navigation systems can be used to localize an aerial system. The most commonly used is inertial navigation (INS or IMU). As this system uses an iterative scheme of accelerations integration, it derives and needs to be updated using additional measurements, such as GNSS. However, most of these systems are exteroceptive, which makes them inoperative in case of jamming. In such a case, terrain navigation, i.e. measurements of the relative elevation, can provide convenient updating information. In this paper, the following scenario is considered: INS has derived and navigation errors must be estimated using only the terrain elevation measurements (for example provided by a radar-altimeter). However, several issues need to be solved: the terrain profile can be strongly non-linear and very ambiguous, and the relationship between measurements and state is difficult to express. One could consider some typical filtering methods such as Extended Kalman Filter (EKF), but these approaches require a local linearization of the terrain measurement that is not robust to strong non-linearity. Multi-hypothesis filtering methods, such as Particle Filters have proved to be able to handle this issue (Nordlund (2002), Bergman (1999)). However, these approaches rely on probabilistic descriptions of measurement and state errors. This is not suitable when these distributions are unknown. Another way of describing errors is defining bounds or intervals within which these values are assumed to remain (Jaulin, 2009). This presents the advantage of not relying on any Gaussian assumptions and makes it possible to guarantee that the estimate remains within bounds. Set-membership estimation techniques aim at determining the set where the estimate vector must belong, given the associated bounds. Several approaches have been presented either to determine exactly the set, but most of them are dedicated to linear models. For non-linear estimation, a recent approach named Box Particle Filter (BPF) has been introduced (Abdallah et al., 2007). Applications to tracking have been presented in (Gning et al. (2012), Gning et al. (2013), DeFreitas et al. (2016)). The key idea is taking advantage of the set-membership approach's simplicity as well as of the stochastic comprehensiveness of Particle Filters.

In this paper, an adaptation of the original Box Particle Filter is presented and applied to terrain navigation. Improvements of this filter have been developed such as regularization (Musso et al., 2001) based on Kernel Estimation (Silverman, 1986) and optimized resampling to reduce the resulting estimation uncertainty.

The paper is organized as follows. In Section 2, the application context of terrain navigation is presented. In Section 3, the definition of the initial BPF applied in the context of terrain navigation is provided, and the improved version named Box Regularized Particle Filter (BRPF) is described. In Section 4, numerical results are provided for comparison purposes between a conventional SIR Particle Filter (PF), a Box Particle Filter (BPF), and the proposed Box Regularized Particle Filter (BRPF).

2. PROBLEM STATEMENT

The evolution model is that of an aircraft represented by its center of mass. The state vector is defined as $\mathbf{x}_k = \left[x_k \; y_k \; z_k \; V_k^x \; V_k^y \; V_k^z\right]^T$ with (x_k, y_k) the horizontal positions, z_k the altitude, and $V^{x,y,z}$ the velocity components.

Let $d \in \mathbb{N}$ be the state space dimension (here $d = 6$), $d_{cont} \in \mathbb{N}$ the control dimension, and $d_{meas} \in \mathbb{N}$ the measurement space dimension (here $d_{meas} = 1$).

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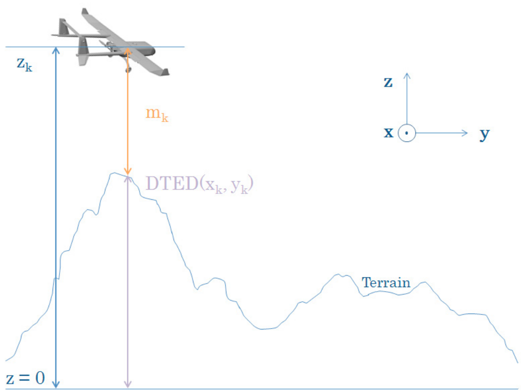

Fig. 1. Elevation measurement in terrain navigation

2.1 Dynamical model

The dynamical model used in this paper is as follows:

$$\mathbf{x}_k = \mathbf{F}\mathbf{x}_{k-1} + \mathbf{B}\mathbf{u}_{k-1} + \mathbf{\Omega}_k \tag{1}$$

where $\mathbf{u}_{k-1} \in \mathbb{R}^{d_{cont}}$ is the control input ($\mathbf{B} \in \mathbb{R}^{d\times d_{cont}}$), and $\mathbf{\Omega}_k \in \mathbb{R}^d$ is the process noise. The distribution $\mathbf{\Omega}_k$ can be unknown. $\mathbf{F}$ is the state evolution matrix.

$$\mathbf{F} = \begin{bmatrix} \mathbf{I}_3 & \mathbf{I}_3\Delta t \\ \mathbf{O}_3 & \mathbf{I}_3 \end{bmatrix} \tag{2}$$

with $\mathbf{I}_3$ the identity matrix and $\mathbf{O}_3$ the zero matrix. The dynamical model can also be more generally expressed as $\mathbf{x}_k = f(\mathbf{x}_{k-1}, \mathbf{u}_{k-1}) + \mathbf{\Omega}_k$ with $f : \mathbb{R}^d \times \mathbb{R}^{d_{cont}} \to \mathbb{R}^d$.

2.2 Measurements

A radar altimeter provides elevation measurements (the relative height m_k, see Fig. 1) along the aircraft trajectory at discrete time values. By comparing on board these elevations with a Digital Elevation Model ($DEM : \mathbb{R}^2 \to \mathbb{R}$), it is possible to reconstruct the absolute position of the aircraft, if there is enough relevant information in the elevation variation. The DEM gives the absolute elevation as a function of the geographical coordinates (x_k, y_k). The measurement equation is:

$$m_k = g(\mathbf{x}_k) + \zeta_k \tag{3}$$

with $\zeta_k \in \mathbb{R}$ the measurement noise, and

$$\begin{aligned} g &: \mathbb{R}^d \to \mathbb{R}^{d_{meas}} \\ g(\mathbf{x}_k) &= z_k - DEM(x_k, y_k) \end{aligned} \tag{4}$$

is the observation function. There is no analytic description of g, since DEM is obtained from an embedded terrain map (see Fig 2).

3. FILTER DESCRIPTION FOR TERRAIN NAVIGATION

The terrain navigation filter is designed to characterize the posterior conditional distribution of the state with respect to the measurements.

Let $\mathbf{M}_k = \{\mathbf{m}_1, ..., \mathbf{m}_k\}$ be the vector of measurements from time 1 to time k. Using $p(\mathbf{x}_{k-1}|\mathbf{M}_{k-1})$, one can deduce the state estimation $\hat{\mathbf{x}}_k = E\left[\mathbf{x}_k|\mathbf{M}_k\right]$ where $E[.]$ is the expectation.

The state estimation consists in two steps: prediction and correction. The prediction step determines a predictive distribution $p(\mathbf{x}_k|\mathbf{M}_{k-1})$ with respect to the dynamical model uncertainty $p(\mathbf{x}_k|\mathbf{x}_{k-1})$ and the previous conditional distribution $p(\mathbf{x}_{k-1}|\mathbf{M}_{k-1})$ via the Chapman-Kolmogorov equation:

$$p(\mathbf{x}_k|\mathbf{M}_{k-1}) = \int p(\mathbf{x}_k|\mathbf{x}_{k-1})p(\mathbf{x}_{k-1}|\mathbf{M}_{k-1})\mathrm{d}\mathbf{x}_{k-1} \tag{5}$$

The correction step determines the posterior conditional distribution of the state with respect to the predictive distribution (5) and the likelihood $p(\mathbf{m}_k|\mathbf{x}_k)$. From the Bayes law, one obtains:

$$p(\mathbf{x}_k|\mathbf{M}_k) = \frac{p(\mathbf{x}_k|\mathbf{M}_{k-1})p(\mathbf{m}_k|\mathbf{x}_k)}{\displaystyle\int p(\mathbf{x}_k|\mathbf{M}_{k-1})p(\mathbf{m}_k|\mathbf{x}_k)\mathrm{d}\mathbf{x}_k} \tag{6}$$

Navigation filters are designed to provide a suitable approximation of the above expressions. Typical filtering methods such as Extended Kalman Filter (EKF) could be used, but these approaches require a local linearization of the terrain measurement function that is not robust to strong non-linearity and usual inertial errors magnitude. Several approaches have been developed to tackle this issue using Particle Filters (Nordlund (2002), Bergman (1999)). However, these methods require to assume probabilistic descriptions of uncertainties (in particular, the measurement noise), that can be unknown.

A potential way of handling this difficulty is to use set membership formalism where the process noise and measurement uncertainty are only assumed to belong to *a priori* known intervals. The estimation consists in characterizing the set of all state vectors that are consistent with the intervals and the model structures. Numerous methods have been developed in this context since the pioneering work of Schweppe (Schweppe, 1968) but a large number is dedicated to linear models. For nonlinear estimation, an interesting approach named Box Particle Filter (BPF) has been recently proposed. Its basic features will be first recalled and the new developments will be described.

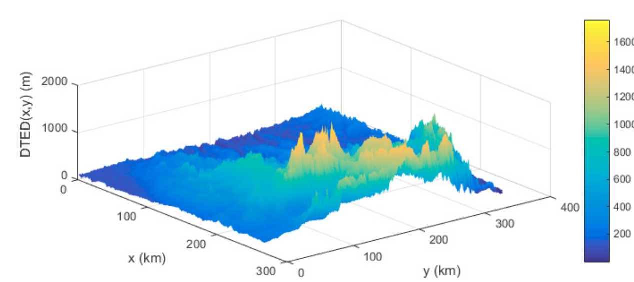

Fig. 2. Example of DEM measurements (Dordogne)

3.1 Box Particle Filter (BPF)

The BPF has been introduced in (Abdallah et al., 2007), and is a translation of the conventional SIR Particle Filter (PF) into the set-membership theory (Milanese et al., 1996). The PF draws a cloud of possible state particles to explore the state space and approximate the conditional distribution with a set of Dirac points. In order to explore the state space more exhaustively, the BPF draws a set of box particles consisting of intervals on each dimension. Therefore, equations have to be adapted according to interval analysis theory. The state vector