Available online at www.sciencedirect.com

ScienceDirect

IFAC-PapersOnLine 49-17 (2016) 373–378

Linearized Impulsive Fixed-Time Fuel-Optimal Space rendezvous: A New Numerical Approach

Denis Arzelier *, **Florent Bréhard** *, **Norbert Deak** *,
Mioara Joldes *, **Christophe Louembet** ***,
Aude Rondepierre **, **Romain Serra** *,

* *CNRS, LAAS, 7 avenue du colonel Roche, F-31400 Toulouse, France and Univ de Toulouse, LAAS, F-31400 Toulouse, France (e-mails: arzelier@laas.fr, florent.brehard@laas.fr, joldes.laas.fr, romain.serra@laas.fr,).*
** *Institut de Mathématiques de Toulouse, Univ. de Toulouse; INSA; F-31062 Toulouse, France, (email: aude.rondepierre@insa-toulouse.fr).*
*** *CNRS, LAAS, 7 avenue du colonel Roche, F-31400 Toulouse, France and Univ de Toulouse, UPS, F-31400 Toulouse, France (e-mail: christophe.louembet@laas.fr).*

Abstract: This paper focuses on the fixed-time minimum-fuel rendezvous between close elliptic orbits of an active spacecraft with a passive target spacecraft, assuming a linear impulsive setting and a Keplerian relative motion. Following earlier works developed in the 1960s, the original optimal control problem is transformed into a semi-infinite convex optimization problem using a relaxation scheme and duality theory in normed linear spaces. A new numerical convergent algorithm based on discretization methods is designed to solve this problem. Its solution is then used in a general simple procedure dedicated to the computation of the optimal velocity increments and optimal impulses locations. It is also shown that the semi-infinite convex programming has an analytical solution for the out-of-plane rendezvous problem. Different realistic numerical examples illustrate these results.

Keywords: Impulsive optimal control, elliptic rendezvous, primer vector, semi-infinite convex programming, discretization methods

1. INTRODUCTION

Since the first space missions (Gemini, Apollo, Vostok) involving more than one vehicle, space rendezvous between two spacecraft has become a key technology raising relevant open control issues. Formation flight (PRISMA), on-orbit satellite servicing or supply missions to the International Space Station (ISS) are all examples of projects that require adequate rendezvous planning tools. A main challenge is to achieve autonomous far range rendezvous on elliptical orbits while preserving optimality in terms of fuel consumption. In short, the far range rendezvous is an orbital transfer between an active chaser spacecraft and a passive target spacecraft, with specified initial and final conditions, over a fixed or a free time period. Searching for the guidance law that achieves the maneuver with the lowest possible fuel consumption leads to define a minimum-fuel optimal control problem.

In this article, the fixed-time linearized fuel-optimal impulsive space rendezvous problem as defined in Carter and Brient (1995), is studied assuming a linearized Keplerian relative motion. The impulsive approximation for the thrust means that instantaneous velocity increments are applied to the chaser whereas its position is continuous. Indirect approaches, based on the optimality conditions derived from the Pontryagin's maximum principle and leading to the so-called primer vector theory (Lawden (1963)), have been extensively studied. For a fixed number of impulses, necessary and sufficient conditions can be derived (Carter and Brient (1995)). However due to the nonconvex and polynomial nature of these conditions, a numerical solution is still difficult to compute and would only be suboptimal for the original rendezvous problem for which the number of possible maneuvers is free. In Arzelier et al. (2013), a mixed iterative algorithm combines variational tests with sophisticated numerical tools from algebraic geometry to solve these polynomial necessary and sufficient conditions of optimality and avoid the local optimization step. However, this algorithm remains heuristic with no proof of convergence in all cases and may exhibit only suboptimal solutions on some instances.

Neustadt (1964) proposed an important theoretical contribution for the optimal control problem: it is recast to a semi-infinite optimization problem, using a relaxation scheme and the duality theory in minimum-norm problems. Claeys et al. (2013) revisit his approach from the angle of generalized moment problems, by formulating it as a linear programming problem on measures. In this approach, the numerical solving is rather cumbersome since one needs high degree polynomial approximations for building hierarchies of linear-matrix inequalities (LMIs). Also, they consider only the case of ungimbaled identical thrusters, which gives a linear problem.

Following Neustadt (1964), we propose a new numerical algorithm to solve the fixed-time impulsive linear rendezvous without fixing *a priori* the number of impulses, and whose convergence is rigorously shown. Firstly, we focus on the moment problem formulation (Sec. 2) and recall topological duality theory results from Luenberger (1969) and Neustadt (1964), which allow for the moment problem to be transformed into a Semi-Infinite Convex Programming (SICP) Problem (Sec. 4). The novelty of our approach is to use discretization methods Reemtsen and Rückman (1998) to solve the SICP problem. A convergent numerical algorithm is designed in Sec. 4,

2405-8963 © 2016, IFAC (International Federation of Automatic Control) Hosting by Elsevier Ltd. All rights reserved.
Peer review under responsibility of International Federation of Automatic Control.
10.1016/j.ifacol.2016.09.064

whose solution is the optimal primer vector of the original rendezvous problem. An estimation of the numerical error made on the optimal cost of the original problem, is also provided. Then, the optimal impulses location and the optimal velocity increments are retrieved via a simple procedure fully exploiting results stated in Neustadt (1964). The efficiency of the proposed algorithm is illustrated with two different realistic numerical examples.

Notations: a, e, ν are respectively the semi-major axis, the eccentricity and the true anomaly of the reference orbit. N is the number of velocity increments while ν_i, $i = 1, \cdots, N$, define impulses application locations. The velocity increment at ν_i will be denoted by $\Delta V(\nu_i)$. $\{b_i\}_{i=1,\cdots,N}$ is a sequence of variables b_i, $i = 1, \cdots, N$, and $\mathrm{sgn}(z)$ is the sign function of the variable z. The prime denotes differentiation with respect to the true anomaly ν. $\mathbf{O}_{p\times m}$ and $\mathbf{1}_m$ denote respectively the null matrix of dimensions $p \times m$ and the identity matrix of dimension m. Let $r \in \mathbb{N}^*$ and $(p, q) \in \mathbb{R}^2$ such that: $1 \le p \le \infty$ and $\frac{1}{p} + \frac{1}{q} = 1$. Classically, $C([\nu_0, \nu_f], \mathbb{R}^r)$ is the Banach space of continuous functions $f : [\nu_0, \nu_f] \to \mathbb{R}^r$ equipped with the norm $\|f\|_q = \sup_{\nu_0 \le \nu \le \nu_f} \|f(\nu)\|_q$. Denote by $\mathcal{L}_{1,p}([\nu_0, \nu_f], \mathbb{R}^r)$ the normed linear space of Lebesgue integrable functions from $[\nu_0, \nu_f]$ to $\mathbb{R}^r$ with the norm given by: $\|u\|_{1,p} = \int_{\nu_0}^{\nu_f} \|u(\nu)\|_p \mathrm{d}\nu$. Let $\mathrm{BV}([\nu_0, \nu_f], \mathbb{R}^r)$ be the space of functions of bounded variation over the interval $[\nu_0, \nu_f]$ with the norm: $\|g\|_{tv,p} = \sup_{P_\kappa} \sum_{i=1}^{\kappa} \|g(\nu_i) - g(\nu_{i-1})\|_p$, where the supremum is taken over all finite partitions $P_\kappa = (\nu_i)_{i=1,\ldots,\kappa}$ of $[\nu_0, \nu_f]$. For a symmetric real matrix $S \in \mathbb{R}^{n\times n}$, the notation $S \preceq 0$ ($S \succeq 0$) stands for the negative (positive) semi-definiteness of S. Finally, χ_A is the indicator function of the set A.

2. PROBLEM STATEMENT AND PRELIMINARIES

This section first introduces and reviews notations and assumptions for the minimum-fuel linearized fixed-time rendezvous problem. Then, adopting the approach of Neustadt (1964), the usual optimal control formulation of the rendezvous problem is recast as a moment problem defined on the functional space $\mathcal{L}_{1,p}([\nu_0, \nu_f], \mathbb{R}^r)$.

2.1 Optimal control formulation of the rendezvous problem

Typically, in a rendezvous situation, a spacecraft is in sufficiently close proximity to allow for the linearization of the relative equations of motion. Their validity is guaranteed when the distance between the target and the chaser is assumed to be small compared to the radius of the target vehicle orbit. The equations of relative motion are written in a moving Local-Vertical-Local-Horizontal (LVLH) frame located at the center of gravity of a passive target and which rotates with its angular velocity. In this frame, the state vector $X^T = [\, p_x \; p_y \; p_z \; v_x \; v_y \; v_z \,]$ is composed of the positions and velocities of a chaser satellite in the in-track, cross-track and radial axes, respectively. Under the previous assumptions and using the true anomaly of the target-vehicle orbit as the independent variable, a system of linear differential equations with periodic coefficients is easily obtained and the considered minimum-fuel linearized rendezvous problem may be reformulated as the following optimal control problem:

Problem 1. (Optimal control problem)
Find $\bar{u} \in \mathcal{L}_{1,p}([\nu_0, \nu_f], \mathbb{R}^3)$ solution of the optimal control problem:

$$\begin{aligned} &\inf_u \|u\|_{1,p} = \inf_u \int_{\nu_0}^{\nu_f} \|u(\nu)\|_p \mathrm{d}\nu \\ &\text{s.t. } X'(\nu) = A(\nu)X(\nu) + B(\nu)u(\nu), \ \forall\, \nu \in [\nu_0, \nu_f] \\ &\qquad X(\nu_0) = X_0, \ X(\nu_f) = X_f \in \mathbb{R}^6, \ \nu_0, \ \nu_f \text{ fixed}, \end{aligned} \tag{1}$$

where matrices $A(\nu)$ and $B(\nu)$ define the state-space model of relative dynamics given by Tschauner (1967):

$$A(\nu) = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 2 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3/(1+e\cos(\nu)) & -2 & 0 & 0 \end{bmatrix}, \quad B(\nu) = \begin{bmatrix} \mathbf{O}_{3\times 3} \\ \dfrac{\mathbf{1}_3}{1+e\cos(\nu)} \end{bmatrix} \tag{2}$$

The form of these matrices shows that the equations describing motion in the plane of the target-vehicle orbit and those describing motion normal to the orbit plane can be decoupled and handled separately. Therefore, the out-of-plane and in-plane rendezvous can be dealt with independently: the state vector dimension and the number of inputs in (1) are denoted n and r, respectively with $n = 2$, $r = 1$ for the out-of-plane case and $n = 4$, $r = 2$ for the in-plane case. Due to space limitations, this paper focuses on the in-plane rendezvous.

Remark 1. In Problem 1, the 1-norm cost captures the consumption of fuel used. In fact, the performance index used in Problem 1 has been normalized to stick to the usual characteristic velocity expressed in m/s.

2.2 A minimum norm moment problem

Following the approach from Neustadt (1964), Problem 1 is now transformed into an equivalent problem of moment by integrating equation (1). As $A \in \mathcal{C}(\mathbb{R}, \mathbb{R}^{n\times n})$, the equation (1) has a unique solution that exists for every $X_0 \in \mathbb{R}^n$ and for all $\nu \in \mathbb{R}$ and for $u(\nu) \in \mathcal{L}_{1,p}([\nu_0, \nu_f], \mathbb{R}^r)$, (Antsaklis and Michel (2003)):

$$X(\nu) = \Phi(\nu, \nu_0)X_0 + \int_{\nu_0}^{\nu} \Phi(\nu, \sigma)B(\sigma)u(\sigma)\mathrm{d}\sigma, \tag{3}$$

where $\Phi(\nu, \nu_0) = \varphi(\nu)\varphi^{-1}(\nu_0)$ and $\varphi(\nu)$ are respectively the transition and Yamanaka-Ankersen fundamental matrices of Keplerian relative motion Yamanaka and Ankersen (2002). Let us define the matrix $Y(\nu) = \varphi^{-1}(\nu)B = [\, y_1(\nu) \cdots y_n(\nu) \,]^T \in \mathbb{R}^{n\times r}$, then:

$$\begin{aligned} c &= \varphi^{-1}(\nu_f)X(\nu_f) - \varphi^{-1}(\nu_0)X_0 \\ &= \int_{\nu_0}^{\nu_f} \varphi^{-1}(\sigma)B(\sigma)u(\sigma)\mathrm{d}\sigma = \int_{\nu_0}^{\nu_f} Y(\sigma)u(\sigma)\mathrm{d}\sigma. \end{aligned} \tag{4}$$

It is important to notice for the remainder of the analysis that for the specific matrices $Y(\nu)$ encountered in the rendezvous problem, $y_1(\nu) \cdots y_n(\nu)$ are linearly independent elements of $\mathcal{C}([\nu_0, \nu_f], \mathbb{R}^r)$. This will be assumed in the rest of the paper. It follows from (4) that Problem 1 can be equivalently written as:

Problem 2. (Minimum norm moment problem) Find $\bar{u}(t) \in \mathcal{L}_{1,p}([\nu_0, \nu_f], \mathbb{R}^r)$ solution of the minimum norm moment problem:

$$\begin{aligned} &\inf_u \|u\|_{1,p} = \inf_u \int_{\nu_0}^{\nu_f} \|u(\nu)\|_p \mathrm{d}\nu \\ &\text{s.t. } \int_{\nu_0}^{\nu_f} Y(\sigma)u(\sigma)\mathrm{d}\sigma = c, \ \nu_0, \ \nu_f \text{ fixed}. \end{aligned} \tag{5}$$

It is well-known that Problem 2 may not reach its optimal solution due to concentration effects (see the reference