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Preview Feedforward Compensation: LMI Synthesis and Flight Simulation

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Abstract: This paper proposes a novel synthesis technique for discrete-time preview feedforward compensation via linear matrix inequality (LMI). While an LMI synthesis condition for preview control in the previous research yields both feedback gain and preview feedforward compensation simultaneously, the proposed approach can derive feedforward compensation solely after feedback gain is designed in some way. LMI synthesis conditions for preview feedforward compensation are obtained based on H_2, H_∞ and IP (impulse-to-peak) performance indices. Simulation results of disturbance attenuation using a research aircraft model are provided to show effectiveness of the proposed synthesis.

Keywords: Preview control, Aircraft control, Linear matrix inequality (LMI).

1. INTRODUCTION

A control methodology that takes advantage of prior information of imminent disturbances or commands is called "Preview control." The idea of preview control dates from Sheridan (1966) and many studies were conducted for both continuous-time systems (Bender (1968); Tomizuka (1975)) and discrete-time systems (Tomizuka (1976); Katayama et al. (1985)). More recently, theoretical analysis of preview control (Kojima (2015)) and applications to active vehicle suspension (Li et al. (2014)) and blade pitch control (Laksz et al. (2010)) have been carried out.

This paper deals with preview control for discrete-time systems. Especially, numerical synthesis of preview feedforward compensation via LMIs (Linear Matrix Inequalities) is focused on. Takaba (2000) presented a numerical synthesis of discrete-time preview control system via LMIs, by which a feedback gain and a preview feedforward compensation are designed simultaneously. On the other hand, it is assumed in this paper that a feedback gain is given beforehand and only a preview feedforward gain needs to be designed. A new augmented system that describes a preview feedforward compensation is introduced, and then the new LMI synthesis conditions are derived in which a feedforward gain is a decision variable without any linearizing change of variables. To show effectiveness of the new synthesis condition, simulation results of disturbance attenuation using a research aircraft model are provided.

2. PROBLEM FORMULATION

Consider a discrete-time system described by

$$x(k+1) = Ax(k) + B_u u(k) + B_w w(k) \tag{1}$$

$$z(k) = C_z x(k) + D_{zu} u(k) + D_{zw} w(k) \tag{2}$$

where $x(k) \in \boldsymbol{R}^n$, $u(k) \in \boldsymbol{R}^p$, $w(k) \in \boldsymbol{R}^r$ and $z(k) \in \boldsymbol{R}^m$ are the state vector, control input, disturbance input and controlled output vector at time step k respectively. In this study $u(k)$ and $w(k)$ are assumed to be scalar inputs ($p = r = 1$) for simplicity.

Assume that the disturbance input $w(k)$ is available from current time step k to $k+h$, which is h step ahead. That is to say,

$$x_d(k) = [\, w(k) \;\; w(k+1) \;\; \ldots \;\; w(k+h) \,]^T$$

is assumed to be available at time step k. Then the vector $x_d(k) \in \boldsymbol{R}^{(h+1)}$ and the disturbance input $w(k)$ are described by

$$x_d(k+1) = A_d x_d(k) + B_d w(k+h+1), \tag{3}$$

$$w(k) = C_d x_d(k), \tag{4}$$

$$A_d = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & \ddots & & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & \cdots & 0 & 1 \\ 0 & 0 & \cdots & 0 & 0 \end{bmatrix} \in \boldsymbol{R}^{(h+1)\times(h+1)},$$

$$B_d = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} \in \boldsymbol{R}^{(h+1)\times 1},$$

$$C_d = [\, 1 \;\; 0 \;\; \ldots \;\; 0 \;\; 0 \,] \in \boldsymbol{R}^{1\times(h+1)}.$$

In the previous study (Takaba (2000)) the following augmented system is introduced using (1)–(4)

$$x_a(k+1) = A_a x_a(k) + B_{ua} u(k) + B_{wa} w(k+h+1) \tag{5}$$

$$z(k) = C_a x_a(k) + D_{ua} u(k) + D_{wa} w(k+h+1) \tag{6}$$

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$$x_a(k) = \begin{bmatrix} x(k) \\ x_d(k) \end{bmatrix},\ A_a = \begin{bmatrix} A_a & B_w C_d \\ 0 & A_d \end{bmatrix},$$
$$B_{ua} = \begin{bmatrix} B_u \\ 0 \end{bmatrix},\ B_{wa} = \begin{bmatrix} 0 \\ B_d \end{bmatrix},$$
$$C_a = [\, C_z\ D_{zw} C_d \,],\ D_{ua} = D_{zu},\ D_{wa} = 0.$$

Note that the state variable $x_a(k)$ includes previewable disturbance signals $x_d(k)$. Thus, if a state feedback controller

$$u(k) = K_a x_a(k) = [\, K_b\ K_f \,] \begin{bmatrix} x(k) \\ x_d(k) \end{bmatrix}$$

can be designed for the augmented system (5)(6), it is apparently composed of a state feedback

$$u_{fb}(k) = K_b x(k) \quad (7)$$

and a preview feedforward compensation

$$u_{ff}(k) = K_f x_d(k). \quad (8)$$

In Takaba (2000) a preview controller gain K_a is designed that minimizes the mixed LQ/H^∞ performance. However, this design technique is not available if the feedback gain K_b is given beforehand and only the feedforward gain K_f needs to be designed. In such cases, design conditions for the augmented system (5)(6) are no longer LMIs and therefore hard to be solved.

3. NEW LMI CONDITIONS FOR PREVIEW COMPENSATION

This section gives a synthesis technique for a preview feedforward gain K_f when the feedback gain K_b is given. Firstly a new state space representation for feedforward compensation is proposed. Then an augmented system with preview information is reconstructed and new LMI synthesis conditions are derived that optimize performance indices such as H_2, H_∞ and impulse-to-peak (IP) performance.

3.1 System representation of feedforward compensation

As mentioned above, the feedforward compensation is represented using (3) and (8):

$$x_d(k+1) = A_d x_d(k) + B_d w(k+h+1) \quad (3)$$
$$u_{ff}(k) = K_f x_d(k) \quad (8)$$

This leads to a simultaneous design of K_b and K_f as in Takaba (2000). Now consider the following dual system:

$$\tilde{x}_d(k+1) = A_d^T \tilde{x}_d(k) + K_f^T w(k+h+1) \quad (9)$$
$$u_{ff}(k) = B_d^T \tilde{x}_d(k) \quad (10)$$
$$(\tilde{x}_d(k) \in \boldsymbol{R}^{(h+1)})$$

Since these are SISO systems and therefore they have the same transfer function, equations (9) and (10) can be used instead of (3) and (8).

Note: The feedforward compensation system (3) and (8) are assumed SISO in this paper for symplicity. It should be noted that, even in the MIMO case, the similar form of (9) and (10) can be derived using a similarity transformation (Hamada (2016)).

3.2 Augmented system

The whole system including the feedforward compensation can be described by a new augmented system with the state $x(k)$, previewable disturbance $x_d(k)$ and dual state $\tilde{x}_d(k)$.

Let

$$u(k) = u_{fb}(k) + u_{ff}(k) \quad (11)$$
$$= K_b x(k) + u_{ff}(k) \quad (12)$$

and the feedback gain K_b is given. Substituting (4)(10)(12) into (1)(2) yields

$$\begin{aligned} x(k+1) &= (A + B_u K_b) x(k) + B_w C_d x_d(k) \\ &\quad + B_u B_d^T \tilde{x}_d(k) \\ z(k) &= (C_z + D_{zu} K_b) x(k) + D_{zw} C_d x_d(k) \\ &\quad + D_{zu} B_d^T \tilde{x}_d(k) \end{aligned}$$

Then the whole system can be described by the following augmented system:

$$\begin{bmatrix} x(k+1) \\ x_d(k+1) \\ \tilde{x}_d(k+1) \end{bmatrix} = A_{cl} \begin{bmatrix} x(k) \\ x_d(k) \\ \tilde{x}_d(k) \end{bmatrix} + B_{cl} w(k+h+1),$$
$$z(k) = C_{cl} \begin{bmatrix} x(k) \\ x_d(k) \\ \tilde{x}_d(k) \end{bmatrix},$$

$$A_{cl} = \begin{bmatrix} A + B_u K_b & B_w C_d & B_u B_d^T \\ 0 & A_d & 0 \\ 0 & 0 & A_d^T \end{bmatrix}, B_{cl} = \begin{bmatrix} 0 \\ B_d \\ K_f^T \end{bmatrix},$$
$$C_{cl} = \left[\, C_{zu} + D_{zu} K_b\ \ D_{zw} C_d\ \ D_{zu} B_d^T \,\right]$$

What is important here is that the decision variable K_f is included only in B_{cl}, not in A_{cl}. This means that, in most cases, synthesis conditions can be described in LMI form using standard LMI analysis conditions (see Masubuchi et al. (1998) for example) without any linearizing change of variables.

3.3 Design condition via LMIs

As an example of LMI design conditions, an H_2 synthesis condition for the augmented system that minimizes the H_2 norm from $w(k+h+1)$ to $z(k)$

$$\|H_{zw}\|_2^2 = \sum_{k=0}^{\infty} z^T(k) z(k)$$

is shown below.

[H_2 synthesis] The preview feedforward compensation $u_{ff} = K_f x_d(k)$ minimizes the H_2 norm if there exist matrix variables $X > 0$, Z and K_f that satisfy the following LMI conditions:

minimize $\gamma_2 > 0$ s.t.