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Optimal-Switched H_∞ Robust Tracking for Maneuvering Space Target

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Abstract: For maneuvering target tracking problem, robust filtering is an effective way to gain fast and accurate target trajectory in real time. The $\boldsymbol{H}_\infty$ filter (H_∞F) is a conservative solution with infinite-horizon robustness, leading to excessive cost of filtering optimality and reduction of estimation precision. In order to retrieve the filtering optimality sacrificed by conservativeness of the H_∞F design, in this paper, an optimal-switched filtering mechanism is developed and established on the standard H_∞F to propose an optimal-switched $\boldsymbol{H}_\infty$ filter (OSH_∞F). The optimal-switched mechanism adopts a switched structure that switches filtering mode between optimal and $\boldsymbol{H}_\infty$ robust by setting a switching threshold, and introduces an optimality-robustness cost function (ORCF) to on-line optimize the threshold such that the switching structure can be optimized. In the ORCF, a non-dimensional weight factor (WF) is used to quantify the ratio of the filtering robustness and optimality. As the only tunable parameter in the filter, when the WF is given, the proposed OSH_∞F can obtain the optimal state estimates with filtering optimality and robustness kept at the WF-determined ratio. With the conservativeness of the H_∞F optimized, the developed OSH_∞F can be used as a generalized H_∞F form. A simulation example of space target tracking has demonstrated the superior estimation performance of the OSH_∞F compared with that of Kalman filter and other typical $\boldsymbol{H}_\infty$ filters.

Keywords: The $\boldsymbol{H}_\infty$ filter, optimal-switched filtering, target tracking, robust estimation.

1. INTRODUCTION

Robust filtering is an effective approach for maneuvering target tracking to acquire accurate and precise real-time target trajectory (Bishop et al. 2007), a typical way of which is to use a H_∞ robust filter. The filter is essentially a state estimator via output measurement, which may guarantee the H_∞ norm of transfer function between estimation error and stochastic uncertainties (noises and external disturbances) to be minimized (Shen et al. 1997; Seo et al. 2006). Such H_∞ criterion makes the filter acquire the best estimates of the worst-possible case. Without requirement of the exact statistical properties of random uncertainties, the H_∞ robust filter only needs the energy of noises and outer disturbances bounded. Due to simplicity of assumption, the H_∞ filter has already been applied in many other areas (Soken et al. 2010; Li et al. 2013; Zhong et al. 2010) to cope with the estimation problems with various types of uncertainties.

A problem of H_∞ filter is that the estimation results yielded by minimizing the H_∞ norm criterion are conservative, that is, too much filtering optimality is sacrificed to obtain the robustness. The reason is that the bounds of uncertainties defined in the H_∞ criterion are given with priori values, but it is unrealistic in practice and the uncertainty bounds are difficult to be determined accurately. In order to improve the optimality of the H_∞ filters, the switched H_∞ filter (SH_∞F) is studied recently by Xiong et al. (2010) and Li et al. (2010; 2014). This filtering approach constructs the filter switched between H_∞ robust and optimal filtering mode by judging an inequality condition related with real measurement covariance, so that the filter robustness becomes finite-horizon and the optimality is enhanced from the time span point of view. Allowing for resetting the Kalman gain, the filter can improve the excessive cost of filtering optimality and augment the tracking accuracy of the maneuvering target with fast transient variations.

The switching mechanism actually replaces the uncertainty bounds with a switching inequality. However, it introduces an auxiliary switching parameter to determine the switching threshold (Li et al. 2010; 2014). The value of the parameter varies with system uncertainties and is usually given as a priori. Although that is simpler than to find the uncertainty bounds, mismatch of the given values will remarkably reduce estimation precision of the switched filters. For this reason, it would be desirable if the switching parameter can be obtained on-line optimally, so that the SH_∞F may independent with any system conditions that need a priori value.

To achieve the objective, an optimal-switched H_∞ filter (OSH_∞F) is proposed in this paper. The filter is built upon the H_∞ filter (H_∞F) provided by Theodor et al. (1994) and the switched structure given by Li et al. (2010; 2014), embedded an extra mechanism to on-line calculate optimal value of the switching parameter at each iteration. The extra mechanism is established by optimizing a quadratic optimality-robustness

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cost function (ORCF), which is defined with the weighted sum of the measurement estimation square error and the measurement prediction covariance. The two parts represent robustness and optimality, reflecting the estimation accuracy and precision of the filter, respectively, and their proportional relation is clarified by a designed non-dimensional weight factor (WF). To implement the proposed OSH∞F, WF is the only parameter that needs given to state quantitatively that how much filtering optimality or robustness will be considered in the estimation results. That prevents the decrement of filtering performance from any switching parameter mismatch, and the results can be regarded as optimal in the sense of WF. Independent with any information of system uncertainties to set parameter, the OSH∞F is a generalized version of the standard H∞F.

A numerical example shows the application of the OSH∞F in space target tracking. It indicates that the OSH∞F has better tracking performances than the usual H∞F and SH∞F, and demonstrates that the filtering results depend on the WF only.

2. SPACE TARGET TRACKING MODEL

For a free-flying orbital chaser and a space target, the relative motion model can be described by (Xu 2005)

$$\ddot{\boldsymbol{R}} = -2\boldsymbol{\omega}_C \times \dot{\boldsymbol{R}} - \dot{\boldsymbol{\omega}}_C \times \boldsymbol{R} - \boldsymbol{\omega}_T \times (\boldsymbol{\omega}_T \times \boldsymbol{R}) + \frac{\mu}{r_C^3} \times \left(\boldsymbol{\rho}_C - \frac{r_C^3}{r_T^3}\boldsymbol{\rho}_T\right) + \boldsymbol{D}^{\boldsymbol{R}} \tag{1}$$

where $\boldsymbol{\rho}$ denote the position in the Earth centered frame (ECF) and $\boldsymbol{R} = [R_x, R_y, R_z]$ the relative position defined in the radial, in-track and cross-track (RIC) frame of the chaser orbit. μ, r and $\boldsymbol{\omega}$ are the geocentric gravitational constant, geocentric distance and orbital angular rate, respectively, and the subscript C or T represents chaser or target. $\boldsymbol{D}^{\boldsymbol{R}}$ describes the external disturbances that include the oblateness perturbation of the Earth and the target maneuver $\boldsymbol{U}_T$ and have bounded energy. With assumption that the relative range is far less than the geocentric distance of the chaser, i.e., $\|\boldsymbol{R}\| \ll r_C$, Equation (1) can be approximated by a discrete-time state equation as (Yoon et al. 2009)

$$\boldsymbol{X}_{k+1} = \boldsymbol{\phi}_{k+1}\boldsymbol{\varphi}_{k+1,k}\boldsymbol{\phi}_k^{-1}\boldsymbol{X}_k + \boldsymbol{D} \tag{2}$$

in which $\boldsymbol{X} = [\boldsymbol{R}, \dot{\boldsymbol{R}}]^T$ is the relative state and $\boldsymbol{D}$ denotes the bounded uncertainties resulted from $\boldsymbol{D}^{\boldsymbol{R}}$. $\boldsymbol{\phi}$ and $\boldsymbol{\varphi}$ are defined by $\boldsymbol{X}_k = \boldsymbol{\phi}_k\delta\boldsymbol{\sigma}_k$ and $\delta\boldsymbol{\sigma}_{k+1} = \boldsymbol{\varphi}_{k+1,k}\delta\boldsymbol{\sigma}_k$ where $\delta\boldsymbol{\sigma}$ is the difference of the orbital element sets of the target and the chaser, i.e., $\delta\boldsymbol{\sigma} = \boldsymbol{\sigma}_T - \boldsymbol{\sigma}_C$. Define the orbital element set as $\boldsymbol{\sigma} = [a, u+f, i, e\cos u, e\sin u, \Omega]^T$ where a, e, i, u, f and Ω are the classical elements that represent semi-major axis, eccentricity, inclination, argument of periapsis, true anomaly and longitude of the ascending node, respectively. The detailed form of $\boldsymbol{\phi}$ and $\boldsymbol{\varphi}$ can be found in Gim et al. (2003).

Denote radar measurement of the chaser as $\boldsymbol{Y} = [\rho, \theta_a, \theta_e]^T$ with noise covariance $\boldsymbol{V}^{\boldsymbol{Y}} = [V_\rho^Y, V_a^Y, V_e^Y]^T$, the components of which represent relative range, azimuth and elevation, respectively. The measurement equation can be expressed as

$$\boldsymbol{C}\boldsymbol{Y}_k = \boldsymbol{R}_k + \boldsymbol{v}_k \tag{3}$$

where $\boldsymbol{C}$ is an unbiased conversion matrix which is derived based on the unbiased converted measurement technique (Mo 1998). $\boldsymbol{v}_k$ is the converted noise with covariance $\boldsymbol{V}$. $\boldsymbol{C}$ and $\boldsymbol{V}$ are determined by $\boldsymbol{Y}$ and $\boldsymbol{V}^{\boldsymbol{Y}}$ and the detailed expressions can be found in Duan et al. (2004). The state estimation model for space target tracking is then given by (2) and (3).

3. H_∞ FILTERING

Rewrite the tracking model (2) and (3) as

$$\boldsymbol{X}_{k+1} = \boldsymbol{F}_k\boldsymbol{X}_k + \boldsymbol{G}_k\boldsymbol{w}_k + \boldsymbol{D} \tag{4}$$

$$\boldsymbol{Z}_k = \boldsymbol{H}_k\boldsymbol{X}_k + \boldsymbol{v}_k \tag{5}$$

where $\boldsymbol{F}$, $\boldsymbol{G}$ and $\boldsymbol{H}$ are matrices independent with the system states and $\boldsymbol{F}_k = \boldsymbol{\phi}_{k+1}\boldsymbol{\varphi}_{k+1,k}\boldsymbol{\phi}_k^{-1}$. $\boldsymbol{Z}$ denotes the measurement vector and equals to $\boldsymbol{C}\boldsymbol{Y}_k$. $\boldsymbol{w}$ and $\boldsymbol{v}$ are process and measurement noises assumed white with covariance matrix $\boldsymbol{W}$ and $\boldsymbol{V}$, respectively.

The objective is to find a filter that estimates the state $\boldsymbol{X}_k$ from the measurement $\boldsymbol{Z}_k$ such that the estimation error $\widetilde{\boldsymbol{X}}_{k|k}$ minimizes a H_∞ norm criterion, specifically, a H_∞ error transfer function

$$\boldsymbol{T}_\infty = \left\| \frac{\widehat{\boldsymbol{X}}_{k|k} - \bar{\boldsymbol{X}}_k}{\boldsymbol{X}_k^{\boldsymbol{Z}} - \bar{\boldsymbol{X}}_k} \right\|_\infty \tag{6}$$

where $\boldsymbol{X}^{\boldsymbol{Z}}$ is the pseudo real states built from measurements. In this paper, we define that a variable with sharp hat represents its estimate, with bar for true value and tilde for estimation error. Clearly, the deviation from $\boldsymbol{X}^{\boldsymbol{Z}}$ to the true value of states $\bar{\boldsymbol{X}}_k$ are yielded by system input uncertainties, so minimizing (6) can be specified by satisfying

$$max \frac{\|\widetilde{\boldsymbol{X}}_{k|k}\|_2^2}{\|\boldsymbol{w}_k\|_2^2 + \|\boldsymbol{v}_k\|_2^2 + \|\boldsymbol{D}\|_2^2} \le \gamma^2 \tag{7}$$

for some minimum $\gamma \in \boldsymbol{R}$. The inequality aims to minimizing the maximum energy transfer efficiency from overall input uncertainties to the estimation error, describing a worst-case performance criterion.

A typical solution for state estimation to (4) and (5) constrained by (7) is the H_∞ filter (H∞F) based on game theory (Banavar 1992; Einicke et al. 1999). Derived by using the H_∞ criterion to build the H_∞ filtering gain, the filter resembles the structure of the standard Kalman filter (KF) and can be performed by the steps as follows.

Step 1: *State Prediction*.

One-step prediction state and prediction covariance are

$$\widehat{\boldsymbol{X}}_{k+1|k} = \boldsymbol{F}_k\widehat{\boldsymbol{X}}_{k|k} \tag{8}$$

$$\boldsymbol{P}_{k+1|k} = \boldsymbol{F}_k\boldsymbol{P}_{k|k}\boldsymbol{F}_k^T + \boldsymbol{G}_k\boldsymbol{W}_k\boldsymbol{G}_k^T \tag{9}$$

Step 2: *Gain Adjustment*.

Calculate H_∞ prediction covariance by

$$\boldsymbol{\Sigma}_{k+1|k} = \left(\boldsymbol{P}_{k+1|k}^{-1} - \gamma^{-2}\boldsymbol{I}\right)^{-1} \tag{10}$$

in which γ satisfies (7). $\boldsymbol{I}$ represents the unit matrix.

Step 3: *Measurement Innovation*.