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Stabilized mixed finite element formulations for materially nonlinear partially saturated two-phase media

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Dedicated to Professor Tom Hughes on the occasion of his 60th Birthday

Abstract

Standard and enhanced (BBAR or EAS) low order finite elements applied to the problem of consolidation of two-phase nonlinear continua do not satisfy the LBB condition when the same interpolation functions are used for both displacement and pore pressure fields. Strong spatial pressure oscillations are the main consequence of the LBB condition violation. The class of direct stabilized methods, widely used in the field of fluid mechanics, is a powerful tool to circumvent violation of the aforementioned condition. Three stabilized formulations, designed for the problem of consolidation of fully or partially saturated media are presented: Galerkin/least-squares (GLS) with the least-squares term construction based on the residuum of the local fluid mass conservation equation; the pressure stabilized formulation (FPL) in which the rate of the pore pressure Laplacian (residual free in the incompressibility limit) is added to the fluid mass conservation equation; and, finally, a stabilized formulation in which the stabilization term is constructed based on the rate of the residuum of the local momentum equation. An *h*-convergence study and test problems for the three stabilized schemes are discussed.

Keywords: Stabilized finite elements; Partially saturated two-phase media

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Nomenclature

δ_{ij}	Kronecker's symbol
ε_{ij}	component of total strain tensor
$\boldsymbol{\varepsilon}$	total strain vector with components: $\{\varepsilon_x, \varepsilon_y, \gamma_{xy}, \varepsilon_z, \gamma_{xz}, \gamma_{yz}\}$
$\varepsilon^{\mathrm{p}}_{ij}$	total plastic strain tensor component
$\boldsymbol{\varepsilon}^{\mathrm{p}}$	total plastic strain vector with components: $\{\varepsilon^{\mathrm{p}}_x, \varepsilon^{\mathrm{p}}_y, \gamma^{\mathrm{p}}_{xy}, \varepsilon^{\mathrm{p}}_z, \gamma^{\mathrm{p}}_{xz}, \gamma^{\mathrm{p}}_{yz}\}$
γ^{F}	fluid unit weight
Γ_{u}	part of the boundary with prescribed displacements
Γ_{p}	part of the boundary with prescribed pressure
Γ_{t}	part of the boundary with prescribed boundary tractions
Γ_{q}	part of the boundary with prescribed boundary fluid fluxes
v	Poisson's ratio
σ_{ij}	component of effective stress tensor
$\boldsymbol{\sigma}$	effective stress vector with components: $\{\sigma_x, \sigma_y, \tau_{xy}, \sigma_z, \tau_{xz}, \tau_{yz}\}$
$\sigma^{\mathrm{tot}}_{ij}$	component of total stress tensor
$\boldsymbol{\sigma}^{\mathrm{tot}}$	total stress vector with components: $\{\sigma^{\mathrm{tot}}_x, \sigma^{\mathrm{tot}}_y, \tau^{\mathrm{tot}}_{xy}, \sigma^{\mathrm{tot}}_z, \tau^{\mathrm{tot}}_{xz}, \tau^{\mathrm{tot}}_{yz}\}$
θ	time integration coefficient
$\mathbf{1}$	Kronecker's vector: $\mathbf{1} = (1 \quad 1 \quad 0 \quad 1 \quad 0 \quad 0)^{\mathrm{T}}$
b_i	component of body force vector
$\mathbf{b}$	body force vector
$\mathbf{B}$	strain–displacement operator
c	storage or compressibility coefficient
D_{ijkl}	elastic stiffness tensor
$\mathbf{D}^{\mathrm{ep}}$	tangent elasto-plastic stiffness matrix
$\mathbf{D}^{\mathrm{e}}$	elastic stiffness matrix
E	Young's modulus
E_{oed}	$E_{\mathrm{oed}} = \frac{E(1-v)}{(1+v)(1-2v)}$
e_0	initial void ratio ($e_0 = n/(I-n)$)
$\mathbf{F}_{\mathrm{EXT}}$	vector of external forces
$\mathbf{F}_{\mathrm{INT}}$	vector of internal forces
K	soil bulk modulus
K^{F}	fluid bulk modulus
k_{ij}	component of permeability tensor
$\mathbf{k}$	permeability matrix: $\mathbf{k} = \begin{bmatrix} k_{x'} & 0 & 0 \\ 0 & k_{y'} & 0 \\ 0 & 0 & k_{z'} \end{bmatrix}$
k	permeability coefficient for one-dimensional problems
$k_{x'}$, $k_{y'}$, $k_{z'}$	permeability coefficients in principal permeability axes
n	porosity
N^{u}_i	shape function for displacement field
N^{p}_i	shape function for pore pressure field
p	pore pressure or solid mean pressure
$\bar{p}$	prescribed pore pressure
$\bar{q}$	prescribed fluid flux