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A stitching method for the generation of unstructured meshes for use with co-volume solution techniques

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Abstract

The successful implementation of co-volume time-domain solution techniques requires the use of high quality, smooth dual meshes. The generation of Delaunay–Voronoï diagrams with these properties, for two-dimensional domains of complicated geometrical shape, is considered. In the adopted approach, near-boundary regions are discretised according to prescribed criteria and the remainder of the computational domain is discretised using an ideal mesh. The two meshes are stitched together to provide a consistent mesh for the complete domain. After smoothing, the resulting mesh is found to be of a quality which exceeds that of meshes produced by standard, automatic, unstructured mesh generation methods. Examples, involving electromagnetic wave scattering, are included to demonstrate the computational performance that can be achieved with a co-volume time domain solution algorithm on these meshes.

Keywords: Delaunay triangulation; Voronoï tessellation; Unstructured co-volume mesh generation; Co-volume time domain solution technique; Electromagnetic wave propagation

1. Introduction

The Yee scheme [1–3], for the solution of the Maxwell equations, and the MAC algorithm, for the solution of the Navier–Stokes equations [4], are co-volume solution techniques that exhibit a high degree of computationally efficiency, in terms of both CPU and memory requirements. However, despite the fact that

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real progress has been achieved in unstructured mesh generation methods over the last two decades, such schemes have not generally proved to be effective for simulations involving domains of complex shape. This is due to the difficulties encountered when attempting to generate high quality dual meshes, that are sufficiently smooth, for such problems. In this paper, we will address this difficulty and describe a procedure for generating unstructured dual meshes, suitable for use with co-volume solution techniques, for general two-dimensional domains.

To illustrate the practical usefulness of the procedure, the simulation of two-dimensional electromagnetic wave scattering problems is considered. The Yee scheme is the classical finite difference co-volume time domain solution technique for this class of problems. A basic requirement for the successful implementation of a co-volume scheme is the existence of two high quality, mutually orthogonal, meshes for the problem under consideration. We begin by describing the implementation of a co-volume solution technique in the unstructured mesh context, where the obvious dual mesh choice is the Delaunay–Voronoï diagram. The requirements that must be placed on the dual meshes are derived and the concept of an ideal mesh for the co-volume solution technique is introduced. Standard mesh generation methods are designed to create high quality Delaunay triangulations, but do not attempt to provide a high quality dual Voronoï mesh. For this reason, we split the problem of triangulation of a domain of complicated shape into a set of relatively simple problems of local triangulation. Each local mesh is constructed with properties which are close to those of the ideal mesh and the local triangulations are combined, to form a consistent mesh, by using a stitching algorithm. The quality of the stitched mesh is improved by the use of standard mesh quality enhancement methods. The usefulness of the proposed procedure is investigated by undertaking electromagnetic wave scattering simulations with both the co-volume technique and an explicit finite element procedure, on meshes produced by different mesh generation methods. It is demonstrated that the co-volume algorithm, implemented on meshes generated by the stitching method, is a significantly more effective computational procedure.

2. Co-volume solution techniques on unstructured meshes

2.1. A co-volume scheme for electromagnetics

We demonstrate the characteristics required of a mesh by constructing initially a co-volume solution technique for electromagnetics. Consider the simulation of two-dimensional problems involving the propagation of transverse electric waves through a medium of permittivity ε and permeability μ. With respect to a Cartesian (x, y, z) co-ordinate system, we assume that the electric field $\boldsymbol{E}$ lies in the (x, y) plane, with the magnetic field $\boldsymbol{H}$ in the z direction. A co-volume scheme requires two mutually orthogonal meshes and we choose to employ the Delaunay–Voronoï dual diagram.

The governing equations are considered in the integral, time domain, form of the laws of Ampère and Faraday [5]. A suitable starting point for the development of the solution algorithm is then to express the laws in terms of integrals over the edges of the Delaunay and Voronoï cells. To illustrate the process, consider a triangular element m of the Delaunay mesh. This element will share an edge with N_m elements, with numbers m_i, $1 \leqslant i \leqslant N_m$, where $N_m = 3$, unless the element has an edge representing the boundary of the domain. Suppose the Delaunay edge mm_i is the common edge between elements m and m_i and let the length of this edge be denoted by ℓ_{mm_i}. Similarly, suppose that the Voronoï edge mm_i is the line segment connecting the circumcentres of element m and element m_i. The length of this Voronoï edge will be denoted by h_{mm_i}. As basic unknowns in the solution algorithm, we consider the value of the z-component of the magnetic field at the Voronoï vertices, and denote this by H_m, and the projection of the electric field at the midpoint of the Delaunay edge mm_i, in the direction of the edge, and denote this by E_{mm_i}. In this case, the laws of Ampère and Faraday can be approximated, using central differencing, as