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Stabilization of systems with interval time-varying delay based on delay decomposing approach

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ABSTRACT

The paper considers the stabilization for systems with interval time-varying delay. By decomposing the delay interval into multiple equidistant subintervals and considering the triple integral terms, a novel Lyapunov-krasovskii functional(LKF) is defined. Then extended integral inequality and convex combination approach are used to estimate the derivative of the constructed functional, and as a result, the new stability criterion with less conservatism and decision variables is obtained. On this basis, the state feedback controller is designed, by using linearization method, the existence condition of controller is obtained in terms of linear matrix inequalities(LMIs), and the specific form of controller is also given, moreover, by selecting the appropriate parameter value, the stabilization time of the system can be reduced. Numerical examples are given to illustrate the effectiveness of the proposed method.

1. Introduction

Time-varying delays are inevitably encountered in practical control systems, such as communication systems, biological systems, nuclear reactors, process control and networked control systems. Since time-varying delays often result in poor performances and even instability, so the stability analysis and controller design of time-varying delay systems have received considerable attention in the last years [1–10].

In order to reduce the conservatism of the stability results, many approaches were developed, among them delay-partitioning approach was an effective method. In [11], by decomposing the discrete and distributed delays into multiple non-uniform subintervals, and constructing suitable Lyapunov-Krasovskii functional in the corresponding interval, the stability criterion was derived. In Li et al. [12], based on the delay-partitioning approach and the Jensen integral inequality, the sufficient conditions for the stability of uncertain neutral systems with mixed delays were given. These methods divided the time delay interval into several parts, with the increased subintervals, the conservatism of the results was reduced, but the complexity of the calculation increased. In order to reduce the computational burden, in Liu et al. [13], by partitioning the delay interval into two segments, constructing suitable Lyapunov-Krasovskii functional, using Wirtinger inequality and reciprocally convex approach, the robust stabilization problem of uncertain time-varying delay system was studied. In Liu et al. [14], by developing a delay decomposition approach and using free matrix, in Cheng et al. [15], by dividing time delay into two subintervals and using free matrix, in Shi et al. [16], by developing a delay decomposition approach and using extended integral inequality, the stability of some different time-varying delay systems were studied, but the constructed Lyapunov-Krasovskii functionals were simple, which caused the conservatism of the main results.

Motivated by above discussions, we further study the stability and stabilization problem of systems with interval time-varying delay. In order to reduce the conservatism of stability condition, we divide the delay interval into N parts with equal length, and then construct a Lyapunov-Krasovskii functional with triple integral terms. By using extended integral inequality and convex combination approach, the less conservative stability conditions are proposed, which also have less decision variables. Then, the feedback controller is designed, and by using linearization method, the existence condition of controller is obtained as well as the specific forms of controller, and by selecting the appropriate parameter value, the stable-time of the system can be reduced. Finally, numerical examples are used to compare with some previous results and demonstrate the effectiveness of the proposed method.

Throughout the note, the used notations are standard. $\mathbf{R}^n$ denotes the n-dimensional Euclidean space, $\mathbf{R}^{n\times m}$ is a set of $n \times m$ real matrix, A^T is the transpose of A, $P > 0 (P < 0)$ means symmetric positive (negative) definite matrix, and $*$ in the matrix denotes the

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symmetric element, I is the identity matrix of appropriate dimensions, $x_t = x(t+\theta)$, $\theta \in [-h, 0]$. Matrices, if their dimensions are not explicitly stated, are assumed to be compatible for algebraic operations.

2. Problem formulation

Consider the following system with interval time-varying delay:

$$\begin{cases} \dot{x}(t) = Ax(t) + A_1 x(t-h(t)) \\ x(t) = \phi(t),\ t \in [-h_D, 0] \end{cases} \tag{1}$$

where $x(t) \in \mathbf{R}^n$ is the state vector; The initial condition $\phi(t)$ is a continuously differentiable vector-valued function; $A, A_1 \in \mathbf{R}^{n\times n}$ are known real constant matrices; $h(t)$ is the time-varying delay satisfying

$$0 \le h_d \le h(t) \le h_D,\ \dot{h}(t) \le \mu$$

where h_d, h_D and μ are constants.

To obtain the main results, the following lemmas are needed.

Lemma 1. *([16]). For any constant matrix $R = R^T > 0$, a scalar $h > 0$, and a vector-valued function $\dot{x}: [-h, 0] \to \mathbf{R}^n$, such that the following integrations are well defined, then the following integral inequality holds*

$$-\int_{t-h}^{t} \dot{x}^T(s)R\dot{x}(s)ds \le -\frac{2}{h}\varsigma_1^T(t)\begin{bmatrix} R & -R \\ -R & R \end{bmatrix}\varsigma_1(t)$$

where $\varsigma_1^T(t) = \begin{bmatrix} \frac{1}{h}\int_{t-h}^{t} x^T(s)ds & x^T(t-h) \end{bmatrix}$.

Lemma 2. *([17]). For any constant matrix $M = M^T > 0$, a scalar $h > 0$, and a vector-valued function $\dot{x}: [-h, 0] \to R^n$, such that the following integrations are well defined, then the following integral inequality holds*

$$-\frac{h^2}{2}\int_{-h}^{0}\int_{t+\theta}^{t} \dot{x}^T(s)M\dot{x}(s)dsd\theta \le \varsigma_2^T(t)\begin{bmatrix} -M & M \\ M & -M \end{bmatrix}\varsigma_2(t)$$

where $\varsigma_2^T(t) = \begin{bmatrix} hx^T(t) & \int_{t-h}^{t} x^T(s)ds \end{bmatrix}$.

Lemma 3. *([18]). Suppose $r_1 \le r(t) \le r_2$, where r_1, r_2 are constants. Then, for any $R = R^T > 0$, and any suitable dimension matrix T_i, $Y_i (i=1,2,3)$, the following integral inequality holds*

$$-\int_{t-r_2}^{t-r_1} \dot{x}^T(s)R\dot{x}(s)ds \le \varsigma_3^T(t)\Big[(r_2 - r(t))TR^{-1}T^T + (r(t) - r_1)YR^{-1}Y^T$$
$$+\begin{bmatrix} Y & -Y+T & -T \end{bmatrix} + \begin{bmatrix} Y & -Y+T & -T \end{bmatrix}^T\Big]\varsigma_3(t)$$

where $\varsigma_3^T(t) = \begin{bmatrix} x^T(t-r_1) & x^T(t-r(t)) & x^T(t-r_2) \end{bmatrix}$, $T = \begin{bmatrix} T_1^T & T_2^T & T_3^T \end{bmatrix}$, $Y = \begin{bmatrix} Y_1^T & Y_2^T & Y_3^T \end{bmatrix}$.

Lemma 4. *([19]). Suppose $\eta_1 \le \eta(t) \le \eta_2$, where η_1, η_2 are constants. Then, for any constant matrices H_1, H_2, H_3 with proper dimensions, the following matrix inequality*

$$H_1 + (\eta_2 - \eta(t))H_2 + (\eta(t) - \eta_1)H_3 < 0$$

holds, if and only if

$$\begin{cases} H_1 + (\eta_2 - \eta_1)H_2 < 0 \\ H_1 + (\eta_2 - \eta_1)H_3 < 0 \end{cases}$$

3. Stability analysis

In this section, the stability of system (1) is investigated. Through dividing the delay interval into $N > 0$ segments and constructing a novel Lyapunov-krasovskii functional, the new stability condition is given by using convex combination approach.

Firstly, the delay interval $\begin{bmatrix} h_d & h_D \end{bmatrix}$ is divided into N equidistant subintervals, $N > 0$ be an integer and $h_i (i = 1, 2, \ldots, N+1)$ be some scalars satisfying: $h_d = h_1 < h_2 < \cdots < h_N < h_{N+1} = h_D$, so the length of every subinterval is $h_\alpha = h_{i+1} - h_i = (h_D - h_d)/N$.

Theorem 1. *For the given scalars $0 \le h_d \le h_D$, μ, system (1) is asymptotically stable if there exist definite matrix $P = \begin{bmatrix} P_{11} & P_{12} & P_{13} \\ * & P_{22} & P_{23} \\ * & * & P_{33} \end{bmatrix} > 0$, $Q_1, Q_2, Q_3 > 0$, $R_1, R_2 > 0$, $U_1, U_2 > 0$ and free matrices $T_1, T_2, T_3, Y_1, Y_2, Y_3$ with appropriate dimensions such that the following LMIs hold*

$$\begin{bmatrix} \Xi & A_c^T S & h_\alpha Y \\ * & -S & 0 \\ * & * & -h_\alpha R_2 \end{bmatrix} < 0, \qquad i = 1, 2, \ldots, N \tag{2}$$

$$\begin{bmatrix} \Xi & A_c^T S & h_\alpha T \\ * & -S & 0 \\ * & * & -h_\alpha R_2 \end{bmatrix} < 0, \qquad i = 1, 2, \ldots, N \tag{3}$$

where

$$\Xi_{11} = P_{11}A + A^T P_{11} + P_{12} + P_{12}^T + Q_1 + Q_2 + Q_3 - h_i^2 U_1 - h_\alpha^2 U_2,$$
$$\Xi_{12} = -P_{12} + P_{13}, \qquad \Xi_{13} = P_{11}A_1 \quad \Xi_{14} = -P_{13},$$
$$\Xi_{15} = A^T P_{12} + P_{22} + h_i U_1, \quad \Xi_{16} = A^T P_{13} + P_{23} + h_\alpha U_2,$$
$$\Xi_{22} = -Q_1 - 2R_1 + Y_1 + Y_1^T, \quad \Xi_{23} = -Y_1 + T_1 + Y_2^T,$$
$$\Xi_{24} = -T_1 + Y_3^T, \qquad \Xi_{25} = -P_{22} + P_{23}^T + \frac{2}{h_i}R_1, \qquad \Xi_{26} = -P_{23} + P_{33},$$
$$\Xi_{33} = -(1-\mu)Q_3 - Y_2 - Y_2^T + T_2 + T_2^T, \qquad \Xi_{34} = T_3^T - T_2 - Y_3^T,$$
$$\Xi_{35} = A_1^T P_{12}, \qquad \Xi_{36} = A_1^T P_{13}, \Xi_{44} = -T_3 - Q_2 - T_3^T, \qquad \Xi_{45} = -P_{23}^T,$$
$$\Xi_{46} = -P_{33}, \quad \Xi_{55} = -U_1 - \frac{2}{h_i^2}R_1, \qquad \Xi_{56} = 0, \quad \Xi_{66} = -U_2.$$

and

$$S = h_i^2 R_1 + h_\alpha R_2 + \frac{1}{4}h_i^4 U_1 + h_s^2 U_2, \quad A_c = \begin{bmatrix} A & 0 & A_1 & 0 & 0 & 0 \end{bmatrix},$$
$$Y = \begin{bmatrix} 0 & Y_1^T & Y_2^T & Y_3^T & 0 & 0 \end{bmatrix}, \qquad T = \begin{bmatrix} 0 & T_1^T & T_2^T & T_3^T & 0 & 0 \end{bmatrix},$$
$$h_\alpha = h_{i+1} - h_i = (h_D - h_d)/N, \qquad h_s = (h_{i+1}^2 - h_i^2)/2,$$

$$h_i = h_d + \frac{(i-1)(h_D - h_d)}{N}.$$

Proof. When $h(t) \in [h_i, h_{i+1}] (i = 1, 2, \ldots, N+1)$, the LKF is constructed as:

$$V_i(t) = \sum_{j=1}^{4} V_{ij}(t) \tag{4}$$

where

$$V_{i1}(t) = \xi^T(t)P\xi(t)$$
$$V_{i2}(t) = \int_{t-h_i}^{t} x^T(s)Q_1 x(s)ds + \int_{t-h_{i+1}}^{t} x^T(s)Q_2 x(s)ds + \int_{t-h(t)}^{t} x^T(s)Q_3 x(s)ds$$
$$V_{i3}(t) = h_i\int_{-h_i}^{0}\int_{t+\theta}^{t} \dot{x}^T(s)R_1\dot{x}(s)dsd\theta + \int_{-h_{i+1}}^{-h_i}\int_{t+\theta}^{t} \dot{x}^T(s)R_2\dot{x}(s)dsd\theta$$
$$V_{i4}(t) = \frac{h_i^2}{2}\int_{-h_i}^{0}\int_{\theta}^{0}\int_{t+\lambda}^{t} \dot{x}(s)U_1\dot{x}(s)dsd\lambda d\theta + h_s\int_{-h_{i+1}}^{-h_i}\int_{\theta}^{0}\int_{t+\lambda}^{t} \dot{x}(s)U_2\dot{x}(s)dsd\lambda d\theta$$

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