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Research article

Modeling, stability analysis, and computational aspects of some simplest nonlinear fuzzy two-term controllers derived via center of area/gravity defuzzification

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A B S T R A C T

The mathematical models reported in the literature so far have been found using Center of Sums (CoS) defuzzification method only. It appears that no one has found models using Center of Area (CoA) or Center of Gravity (CoG) defuzzification method. Although there have been some works reported to deal with modeling of fuzzy controllers via Centroid method, all of them have in fact used CoS method only. In this paper, for the first time mathematical models of the simplest Mamdani type fuzzy Proportional Integral (PI)/Proportional Derivative (PD) controllers via CoG defuzzification are presented. L-type and Γ-type membership functions over different Universes of Discourse (UoDs) are considered for the input variables. L-type, Π-type and Γ-type membership functions are considered for the output variable. Three linear fuzzy control rules relating all four input fuzzy sets to three output fuzzy sets are chosen. Two triangular norms namely Algebraic Product (AP) and Minimum (Min), Maximum (Max) triangular co-norm, and two inference methods, Larsen Product (LP) and Mamdani Minimum (MM), are used. Properties of the models are studied. Stability analysis of closed-loop systems containing one of these controller models in the loop is done using the Small Gain theorem. Since digital controllers are implemented using digital processors, computational and memory requirements of these fuzzy controllers and conventional (nonfuzzy) controllers are compared. A rough estimate of the computational time taken by the digital computer while implementing any of these discrete-time fuzzy controllers is given. Two nonlinear plants are considered to show the superiority of the simplest fuzzy controller obtained using CoA or CoG defuzzification method over the simplest fuzzy controller obtained using CoS method and reported recently. Real-time implementation of one of the developed controller models is done on coupled tank experimental setup to show the feasibility of the developed model.

1. Introduction

Mathematical modeling and stability analysis of fuzzy control systems have been the topics of interest to many researchers. Mathematical model showing the relationship between the input and output variables is very useful in analysis, synthesis, and implementation of fuzzy controllers. Linear PI/PD controllers do not work satisfactorily for nonlinear and time-varying systems. Nonlinear controllers such as fuzzy PI/PD controllers provide better performance for such systems. Control practitioners prefer such a controller which introduces less computational time-delay and involves minimal cost of replacement of the controllers that are already in use. Mathematical model of a fuzzy controller depends on many factors like membership functions, triangular norms (t-norms, in short), triangular co-norms (also called s-norms sometimes), inference methods, and defuzzification strategies used. Thus, a fuzzy controller does not have a unique model, i.e., different combinations of these factors lead to different models. In this context, choosing a particular model for the implementation on a digital computer is one of the possible choices the practitioner has. Only the existing linear (nonfuzzy) controller expression needs to be replaced by a nonlinear fuzzy controller model expression. Mathematical models of fuzzy controllers are useful in the following: establishing stability conditions for feedback systems that contain one of the controller models in the loop, obtaining optimal values of controller parameters using soft computing techniques like Genetic Algorithms (GAs), studying computational and memory requirements in the implementation of

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controllers on a digital computer, etc. The study on computational and memory requirement aspects helps control practitioners choose the best possible fuzzy controller which produces better performance with less computational time-delay and reasonable cost of implementation.

In the literature, several models of fuzzy controllers were obtained, their properties were studied, their suitability/unsuitability for control was discussed, and conditions for ensuring feedback systems stability were established. We now briefly present some important historical developments in the area of modeling and analysis of the simplest fuzzy PI or PD controllers of Mamdani type. Methods for improving performance of conventional fuzzy PI controller when applied to systems of higher order was investigated [2]. Two types of fuzzy controllers were proposed to remove appropriate amounts of accumulated control input given by fuzzy PI controllers. The proposed method gave reduced rise time as well as small overshoot compared to the results obtained by applying conventional fuzzy PI controllers. Mathematical models of the simplest fuzzy PI controller using different inference methods were first obtained by Ying [3] and this work laid the foundation for the development of fuzzy control theory. The simplest fuzzy controller is a controller which involves minimal number of input and output fuzzy sets. An adaptive tuning of parameters of PI controller using fuzzy logic was proposed [5] to improve the performance of the system. Equivalence between fuzzy logic controllers and PI controllers was shown [6]. Absolute stability conditions for the control system having fuzzy PD controller were derived [7] using Yakubovich's method. Sufficient condition for Bounded-Input Bounded-Output (BIBO) stability of fuzzy PI control systems was established by using the Small Gain theorem [8]. Self-tuning fuzzy PD controller based on two-stage tuning of scaling factors was proposed [9]. Li [10] introduced a methodology for designing and tuning the scaling gains of fuzzy controller based on its well-tuned linear counterpart. A simple method for self-tuning of scaling factors of fuzzy PI controllers was proposed [11]. A novel technique of automatic tuning of a PI controller using fuzzy inference mechanism and dead-beat approach was presented [13]. Xu et al. [14] proposed a tuning method based on gain and phase margins to determine the weighting coefficients of the fuzzy PI controller. A simple and robust method for tuning of output scaling factor of fuzzy PI/PD controller was proposed [15]. Mann et al. [16] investigated different fuzzy PID controller structures. By expressing the fuzzy rules in different forms, each PID structure was distinctly identified. A simple method for online tuning of output scaling factors of fuzzy PI controllers was proposed [17]. The proposed scheme was tested for a wide variety of processes including a marginally stable system with different values of dead time. Analytical structures of various fuzzy controllers were presented in the book [18]. Stability analysis, design and real time implementation of some of the developed analytical structures were nicely discussed in that book.

Mudi and Pal [19] have shown that a fuzzy PI controller with resetting action behaves close to a fuzzy PD controller and may result in a steady state error. Patel and Mohan [20] derived analytical structures of the simplest fuzzy PI controllers using triangular output membership functions, AP and Min t-norms, bounded sum and Max s-norms, 12 different inference methods, four linear fuzzy control rules and CoS defuzzification method. They investigated the properties of such controllers and made a comparative study on the same. Moreover, they established sufficient conditions for BIBO stability of fuzzy PI control systems. Subsequently Ali and Ying [22] developed models by using nonlinear membership functions for the input fuzzy sets. A simple enhanced fuzzy PD controller by incorporating two nonlinear discrete tracking differentiators into a conventional fuzzy PD controller was presented [23]. The main improvement of the enhanced fuzzy PD controller lies in its high robustness against noise and ease of implementation. Ali and Ying [24] studied the analytical structures of two - dimensional and three-dimensional fuzzy controllers by employing nonlinear input fuzzy sets of arbitrary type. Properties of the simplest fuzzy PI controllers obtained via CoG defuzzification were studied through computer simulation by Patel [25]. Mathematical models and the corresponding properties of one-dimensional and two-dimensional fuzzy controllers were investigated by Ban et al. [27]. Fuzzy subtractive clustering method was used to determine the membership functions and rule base of fuzzy PI and PD controllers [28]. They also proposed a method to further reduce the number of membership functions and number of rules using a similarity measure. A class of PI-fuzzy controllers dealing with a class of integral plants to ensure their robust stability in the face of parametric variations of the controlled plant was proposed [29]. A simple and effective method for tuning of fuzzy PI controller based on fuzzy logic was proposed [30]. The input scaling factors are tuned online by gain updating factors whose values are determined by rule base with the error and change in error as inputs according to the required controlled process. A clustering method is used to reduce the fuzzy inference rules to reduce the computational time and memory. A PI-adaptive fuzzy control architecture for a class of uncertain nonlinear systems was proposed [31]. This method aims to provide added robustness in the presence of large and fast but bounded uncertainties and disturbances. Lyapunov analysis is used to prove asymptotic stability of the proposed approach. Mohan and Sinha [32] unveiled mathematical models of the simplest fuzzy PI/PD controllers by employing skewed fuzzy sets, Min/AP t-norm, Max s-norm, and MM/LP inference method.

Auto-tuning of fuzzy PD and PI controllers using reinforcement Q-learning algorithm for single-input single-output and two-input two-output systems was proposed [33]. Analytical structures of interval type-2 fuzzy PI/PD controllers have been derived and their properties in the context of control have been studied [34]. Some shortcomings did take place inadvertently in some of the publications which are actually leading to misunderstandings and confusion. So, Mohan [35] made an attempt to dispel all those misunderstandings. Bosukonda and Kelothu [36] revealed mathematical models of the simplest fuzzy PI/PD controllers by employing equal UoDs for both the scaled input variables; L-type, Π-type and Γ-type membership functions for the scaled output fuzzy sets; LP inference and three linear fuzzy control rules. These models are completely new and qualitatively different from those reported earlier in the literature. A stable adaptive fuzzy controller for a class of nonlinear systems was introduced [37]. The main features of the proposed adaptive fuzzy controller are a fuzzy PD-like system for canceling the nonlinear function and a supervisory controller that essentially acts as an integrator to drive the tracking error toward zero. An approach to implement fuzzy PI and fuzzy PD controllers on a Field Programmable Gate Array (FPGA) was proposed [38]. This implementation improved speed, accuracy, efficiency and cost. Tuning of fuzzy PD cascade controller was done using Particle Swarm Optimisation (PSO) [39]. Tuning of PI and PD controllers for uncertain linear systems was done using fuzzy gain margin and phase margin specifications [40]. Nie and Tan [41] have presented analytical structure and characteristics of symmetric Karnik-Mendel type reduced interval type-2 fuzzy PI and PD controllers. Hušek [42] proposed a robust PI controller design with respect to fuzzy sensitivity margins. Design of fractional fuzzy PD+I controllers was done using the GA [43]. Arun and Mohan [44] obtained mathematical models of the simplest fuzzy two-term controllers using CoS defuzzification. They also studied the computational aspects of the developed controller models. New techniques to calculate the dynamic gains of nonlinear systems represented by fuzzy basis function network