ISA Transactions ∎ (∎∎∎∎) ∎∎∎–∎∎∎

Contents lists available at ScienceDirect

ISA Transactions

journal homepage: www.elsevier.com/locate/isatrans

Research article

Finite time stability and controller design for nonlinear impulsive sampled-data systems with applications ☆

Xiaoxiao Lv [a], Xiaodi Li [a,b,*]

[a] School of Mathematics and Statistics, Shandong Normal University, Ji'nan, 250014, PR China
[b] Institute of Data Science and Technology, Shandong Normal University, Ji'nan, 250014, PR China

ARTICLE INFO

Article history:
Received 12 May 2017
Received in revised form
22 July 2017
Accepted 27 July 2017

Keywords:
Sampled-data control
Nonlinear systems
Finite time stability (FTS)
Impulse
Linear matrix inequality (LMI)

ABSTRACT

This paper investigates the finite time stability (FTS) for nonlinear impulsive sampled-data systems. By constructing an appropriated Lyapunov function and employing average impulsive interval (AII) method, some FTS criteria for the nonlinear impulsive sampled-data systems are derived in terms of linear matrix inequalities (LMIs), which can be easily verified via the LMI toolbox. The hybrid controller including sampled-data controller and impulsive controller is designed via the established LMIs. Moreover, the impulse effect considered in this paper including stabilizing impulse and destabilizing impulse. Our developed results are less conservative than the recent work in the literature. Finally, two numerical examples are provided to show the applications of the proposed criteria.

1. Introduction

During the past decades, finite time stability (FTS) in dynamical systems, have received considerable attentions since it was first introduced in 1950s and it has found applications in practical process, such as, avoiding saturation or the excitation of nonlinear dynamics during the transient [1,2]. FTS is a system property concerning the quantitative behavior of the state variables over an assigned finite-time interval. Given a bound on the initial condition, a system is said to be FTS if its state (weighted) norm does not exceed a certain threshold during the specified time interval. Hence FTS enables us to specify quantitative bounds on the state of a dynamical system and play an important role in addressing transient performances of control systems. Therefore, in recent years, many interesting result for FTS have been proposed, see [3–7] for instances. It should be noticed that FTS and Lyapunov Asymptotic Stability (LAS) are different concepts, indeed, a system can be FTS but not LAS, and vice versa [8–10]. While LAS deals with the behavior of a system within a sufficiently long (in principle, infinite) time interval, FTS is a more practical concept, useful to study the behavior of the system within a finite (possibly short) interval, and therefore it finds application whenever it is desired that the state variables do not exceed a given threshold during the transients, such as robot control, missile systems and so on [11,12].

Recently, many interesting control schemes are proposed to design effective controllers, such as activation feedback control, the linear coupling method, the sliding mode control, the sampled-data control, and the impulsive control, see [16–21]. Different control schemes have different advantages. Especially, sampled-data control method has been studied extensively in the past years. As we know, the analysis of linear control systems is based on the fact that the signals at various points in the system are continuous with respect to time. However, in some applications it is convenient to use one or more control signals at discrete time intervals. The control systems using one or more signals at discrete time intervals are known as sampled-data control systems. In a sampled-data control method, the signal at any one or more places is sampled and appears in the form of a pulse at certain intervals. Compared with continuous controller, the sampled-data controller has many advantages such as easy installation, high reliability, maintenance with low cost, and efficiency. Therefore, sampled-data control systems have been becoming important topic in various research fields [13–15,18]. For example, a new looped-functional-based approach has been proposed in [13] for analyzing the stability of periodic and aperiodic uncertain sampled-data systems with incremental delays. [14] have presented a result on sampled-data-based state feedback stabilization of a class of

☆This work was supported by National Natural Science Foundation of China (11301308, 61673247), and the Research Fund for Distinguished Young Scholars and Excellent Young Scholars of Shandong Province (ZR201702100145, ZR2016JL024). The paper has not been presented at any conference.

* Corresponding author at: School of Mathematics and Statistics, Shandong Normal University, Ji'nan 250014, PR China.
E-mail address: lxd@sdnu.edu.cn (X. Li).

http://dx.doi.org/10.1016/j.isatra.2017.07.025
0019-0578/© 2017 ISA. Published by Elsevier Ltd. All rights reserved.

switched linear neutral systems under asynchronous switching. The problem of sampled-data exponential synchronization has been investigated in [15] for a class of Markovian jumping chaotic neural networks with time delays. It should be mentioned that, there are also some studies on the FTS of sampled-data systems [22,23]. Robust finite-time sampled-data control of linear systems has been studied in [22]. Based on the constructed Lyapunov-Krasovskii functional, Wirtinger integral inequality and linear matrix inequalities (LMIs), some FTS conditions of fuzzy systems have been derived in [23].

On the other hand, the impulse as an effective control method has been widely used in many areas such as orbital transfer of satellite, dosage supply in pharmacokinetics, ecosystems management, see [24–29]. The basic theory for impulsive differential systems has been extensively investigated in the past several years, see [30–36]. Its necessity and importance lie in that, in many cases, a real system may encounter some abrupt changes at certain time moments and cannot be considered continuously. The main idea of impulsive control is to change the states instantaneously at certain instants. Therefore, impulsive control can reduce control cost and the amount of transmitted information drastically. A novel concept of the dual-stage impulsive control and its practical framework were proposed to synchronize a class of chaotic DNNs with different time-varying parametric uncertainties for the first time [38]. The designed dual-stage impulsive controller can not only realize the exponential synchronization with error bound but also estimate the exponential convergence rate. It is worth pointing out that the results presented in this paper provide an important theoretical foundation for impulsive synchronization of multi-perturbation delayed nonlinear systems. Until now, there are many valuable results for FTS of impulsive systems [39–41]. In [39], necessary and sufficient condition for FTS of impulsive dynamical linear systems is proposed. In [40], necessary and sufficient conditions for the input-output finite-time stability of impulsive linear systems were investigated. The literature [41] studied FTS of genetic regulatory networks with impulsive effects. However, to the best of our knowledge, there are few results about the FTS of sampled-data impulsive systems [43]. As we know, the concept of average impulse interval (AII) has been firstly introduced in [42], which is suitable for characterizing a wide range of impulsive signals. As long as the AII constant satisfies certain condition, it is not necessary to impose restrictions on the upper and lower boundary of impulsive intervals. Therefore, AII is an effective tool to deal with the non-uniformly distributed impulses. Based on this method, very recently ref [43] established some FTS criteria for the linear time-invariant sampled-data system with impulsive effects, where the requirement on the upper-lower bounds of the sampled intervals is fully removed. However, the result in [43] is based on the fact that there exists strict restriction on the state of system at the impulsive time t_k, i.e., it assumed that there exists a positive constant $d > 0$ such that $x^T(t_k)Px(t_k) < d$ for all $k = 1, 2, \dots, r$, where d will be used to determine the FTS. Note that in real applications, it is usually difficult to estimate the state of the system. Moreover, the impulsive controller is not designed in the paper. These motivate the present study.

In present paper, the problems of FTS are investigated for a class of nonlinear impulsive sampled-data systems. By constructing an appropriated Lyapunov function and employing AII method, some LMI-based sufficient conditions are derived to guarantee to the FTS of the addressed nonlinear systems. We don't impose any restriction on the state of the system, which improve the result in [43]. The hybrid controller including sampled-data controller and impulsive controller is designed via the established LMIs. The advantage of the hybrid controller is that when a controller is unactivated or activated but the control effect is not so well, the another controller will play positive effect and be helpful to the control of the system. Moreover, our result can be applied to the system subject to destabilizing impulses, that is, when the impulses destroy the dynamics, we can utilizing the sampling-data controller to stabilize the system and achieve the desirable FTS. Thus the impulse effect considered in this paper including stabilizing impulse and destabilizing impulse. The rest of this paper is organized as follows. In Section 2, some notations, definitions and a well-known technical lemma are given. Section 3 presents the main results. As applications, two numerical examples and their computer simulations are provided in Section 4. Finally, the paper is concluded in Section 5.

Notations. Let $\mathbb{Z}_+$ denote the set of positive integer, $\mathbb{R}$ the set of real numbers, $\mathbb{R}_+$ the set of positive numbers, $\mathbb{R}^n$ the n-dimensional real spaces equipped with the Euclidean norm $|\bullet|$ and $\mathbb{R}^{n\times m}$ the $n \times m$-dimensional real spaces. $A > 0$ or $A < 0$ denotes that the matrix A is a symmetric and positive definite or negative definite matrix. The notation A^T and A^{-1} denote the transpose and the inverse of A, respectively. If A, B are symmetric matrices, $A > B$ means that $A - B$ is positive definite matrix. $\lambda_{\max}(A)$ and $\lambda_{\min}(A)$ denotes the maximum eigenvalue and the minimum eigenvalue of matrix A, respectively. I denotes the identity matrix with appropriate dimensions and $\Lambda = \{1, 2, \dots, n\}$. For $J \subseteq \mathbb{R}$ and $S \subseteq \mathbb{R}^k$ with $1 \le k \le n$, let $C(J, S) = \{\varphi: J \to S, \varphi \in C_0\}$ and $\mathcal{F} = \{\varphi: [t_0, \infty) \to P, \varphi \in F_0\}$, where C_0 is the set of continuous functions, and F_0 is the set of continuously differentiable functions. Notation $\star$always denotes the symmetric block in a symmetric matrix.

2. Preliminaries

Consider the following nonlinear system,

$$\dot{x}(t) = Ax(t) + Bf(x(t)) + u(t), \tag{1}$$

where $x(t) = (x_1(t), \dots, x_n(t))^T \in \mathbb{R}^n$ and $u(t) \in \mathbb{R}^n$ denotes the state and the control input, respectively; $A, B \in \mathbb{R}^{n\times n}$; the nonlinearity $f \in \mathbb{C}(\mathbb{R}^n, \mathbb{R}^n)$ satisfies

$$l_i^- \le \frac{f_i(\alpha_1) - f_i(\alpha_2)}{\alpha_1 - \alpha_2} \le l_i^+$$

for all $\alpha_1 \ne \alpha_2$, where $f = (f_1, \dots, f_n)^T$ and for any $i \in \Lambda$, $f_i(0) = 0$, l_i^- and l_i^+ are some real constants and they may be positive, zero or negative.

We consider the hybrid controller in the form of

$$u(t) = K_1x(t_k) + K_2x(t)\delta(t - t_{k+1}), \ t \in [t_k, t_{k+1}), \ k \in \mathbb{Z}_+,$$

where $K_1, K_2 \in \mathbb{R}^{n\times n}$ are the feedback controller gains to be designed, $\delta(\cdot)$ is the Dirac delta function with sequence $\xi = \{t_1, t_2, \cdots\}$ satisfying

$$0 = t_0 < t_1 < t_2 < \cdots < t_k < \cdots, \ \lim_{k\to\infty} t_k = \infty.$$

Then, the system (1) is rewritten as

$$\begin{cases} \dot{x}(t) = Ax(t) + Bf(x(t)) + K_1x(t_k), \ t \in [t_k, t_{k+1}), \\ \Delta x = K_2x(t_k^-), \ k \in \mathbb{Z}_+, \\ x(0) = x_0 \in \mathbb{R}^n, \end{cases} \tag{2}$$

where $\Delta x = x(t_k) - x(t_k^-)$, $x(t_k) = x(t_k^+)$ and $x(t_k^-) = \lim_{t\to t_k^-} x(t)$.

To derive the main results, the following definitions and lemma are introduced.

Definition 1 (*Lu et al.* [42]). The average impulsive interval (AII) of the impulsive sequence $\xi = \{t_1, t_2, \cdots\}$ is equal to τ_α if there exist $N_0 \in \mathbb{Z}_+$ and $\tau_\alpha > 0$ such that