ISA Transactions ∎ (∎∎∎∎) ∎∎∎–∎∎∎

Contents lists available at ScienceDirect

ISA Transactions

journal homepage: www.elsevier.com/locate/isatrans

Research article

Observer-based stabilization for switched positive system with mode-dependent average dwell time

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ARTICLE INFO

Article history:
Received 22 December 2015
Received in revised form
23 May 2017
Accepted 8 July 2017

Keywords:
Mode-dependent average dwell time
Quasi-time-dependent
Observer-based
Switched positive system

ABSTRACT

This paper concerns the exponential stabilization problem for a class of switched positive systems. The switching signal satisfies mode-dependent average dwell time (MDADT) and the state variables are partially unmeasurable. A further explanation of mode-dependent average dwell time is included. By employing a set of quasi-time variables, which is first proposed for switched systems with MDADT, new stability results are obtained for the switched nonlinear systems and the underlying linear systems. Observer-based stabilization controllers, both for single-input case and multi-input case, are designed such that the closed-loop system converges exponentially. The designed observers and controllers are both mode-dependent and quasi-time-dependent, which is proved to be less conservative than the ones only mode-dependent. A simplified design strategy is provided to reduce the computation burden. Illustrative examples are provided to demonstrate the effectiveness of the results.

1. Introduction

Switched systems have been investigated for a few decades due to their effectiveness for modelling the stochastic process in practical applications, such as mass-spring systems [1], LLC DC/DC resonant converter [2], fault-tolerant systems [3], sensor networks [4,5]. Positivity is required in vast of switched systems such as robotics, power electronics, transportation systems [6] and has attracted large amount of attentions (see [7–9] and references therein).

Generally specking, there are three types of switched systems with respect to the switching signal: state-dependent switching [10–12], stochastic switching [13,14] and time-dependent switching [15–20]. The state-dependent switching has been widely used for PID-controller pinning and system identification while Markovian chain is usually employed to describe the stochastic process. As for the time-dependent switching, an arbitrary switching signal is usually employed in literature (such as [17] and references therein) to model the switching mechanism among subsystems. To qualitative analysis the behavior of switched systems with constrained switching signals, many special classes of switching signal are proposed. Three well-known constrained switching signals are dwell time (DT) switching signal [16], average dwell time (ADT) switching signal [19,21] and persistent dwell time (PDT) switching signal [1], which are investigated extensively by researchers. Note that almost all these switching signals are only time-dependent. To further relax the constraints of such switching signals, mode-dependent average dwell time (MDADT) switching signal is first proposed in [20], in which the stability and stabilization problems for switched linear systems are investigated in terms of linear matrix inequality (LMI) approach. Thereafter, filtering problem has been investigated in [22,23] for systems with such switching signal. When referring to the control problems of switched systems with MDADT, one can see [24–26]. Other results for switched systems under MDADT switching can be found in [27–29] and references therein.

A primary problem for switched positive systems with MDADT is stabilization. Generally speaking, state-feedback strategy or output-feedback strategy is utilized to tackle this problem [20,30]. The former is used more widely than the latter because the latter will always lead to a descriptor system, which leaves a more complex problem to be dealt with. Unfortunately, systems in practical applications, such as LLC DC/DC Resonant Converter [2], decentralized networked control systems [31], explicit force system [32], neural networks [33], always have unmeasurable states. In such a case, designing an observer is necessary when employing the state-feedback stabilization strategy. Recently, extended Kalman estimator and sliding mode observer are designed in [34] and [35] respectively. After that, high-gain observers is proposed in [36] to solve the large angle tracking control problem. However, these observers are designed either for non-switching systems or

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http://dx.doi.org/10.1016/j.isatra.2017.07.010
0019-0578/

for stochastic switching systems without taking positivity into consideration. To the best of our knowledge, so far, the most advanced observers designed for time-dependent switched positive system are published in [37,38]. Both of these two previous works are obtained under the consideration of ADT switching signal. They are not suitable for switched positive system with MDADT. Moreover, no similar result has been published for switched positive system with MDADT. In addition, Although the quasi-time-dependent technique has been employed in literature [1] for systems with PDT switching signal successfully, it is still significantly challenging to employ this technique in systems with MDADT switching signal. The reason is that unlike the structure of PDT switching signal, no minimal time interval between any two consecutive switchings can be found for MDADT switching signal at all time. This motivates our current work.

In this paper, new results on observer-based stabilization are worked out for switched positive system with MDADT by taking full use of a set of new defined quasi-time variables. After providing the structure of the considered closed-loop system, stabilization results are given step-by-step. The stability of a kind of switched non-linear system with MDADT is considered firstly. The stability conditions are less conservative than that of in literature because the selected Lyapunov function is not only mode-dependent but also quasi-time-dependent. Afterwards, the stability criteria is extended to switched positive linear autonomous system under MDADT switching, which is followed by a sort of observer realization with fixed type. Then observer-based controller gains, both for single-input systems and for multi-input systems, are designed in a form of both mode-dependent and quasi-time-dependent. Through such an approach, the controller design requires to solve linear programming problem twice. In order to reduced the computation burden, an alternative design method is presented. Finally, numerical examples show that the new designed observer is effective for providing feedbacks to controllers, i.e. the designed controller can stabilize the underlying switched positive linear system with MDADT.

Notations: Notations employed in this paper are standard. **1** is introduced to denote a matrix with all entries are ones and '0' is zero matrix. For simplicity, it is assumed that all the vectors and matrices are with proper dimension if there are no special explanation. For a vector $v = [v_1, v_2, \ldots, v_n]$, diag($v$) stands for a diagonal matrix whose diagonal elements are v_i, $i \in \{1, 2, \ldots, n\}$. If a function κ: $[0, +\infty) \to [0, +\infty)$ is continuous, strictly increasing, unbounded and $\kappa(0) = 0$, then κ is said to be of class κ_∞. A vector or matrix c is said to satisfy $c \succ 0$ (or $\succeq 0$) means that all the entries of c are positive (or nonnegative), while symbols $\preceq$ and $\prec$ stand for the opposite. Operator $\|\cdot\|$ gives the 1-norm of a vector or a matrix.

2. Problem statement and preliminaries

The closed-loop switched positive system considered in this paper is depicted in Fig. 1. It can be seen from the figure that the closed-loop system is composed by three parts: the opened-loop system to be stabilized, the observer and the observer-based stabilization controller to be designed.

Firstly, consider the opened-loop system with m subsystems, from Fig. 1, it can be derived as

$$\begin{cases} x(k+1) = A_{r(k)}x(k) + B_{r(k)}u(k) \\ y(k) = C_{r(k)}x(k) + D_{r(k)}u(k) \end{cases} \tag{1}$$

where $x(k)$ and $y(k)$ stand for the partially unmeasurable state variable and the measurable output of the system, $u(k)$ is the control input. $r(k)$ denotes the switching signal whose value is taken from a finite set $\vartheta = \{1, 2, \ldots, m\}$. $A_{r(k)} \succeq 0$, $B_{r(k)} \succeq 0$, $C_{r(k)} \succeq 0$, $D_{r(k)} \succeq 0$ are

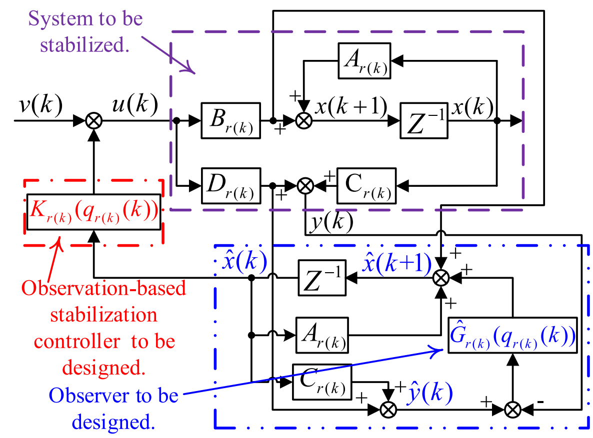

Fig. 1. Block diagram of the closed-loop system considered in this paper.

system matrices of m_{th} subsystems. For simplicity, A_p, B_p, C_p, D_p will be used to denote the system matrices corresponding to subsystem $p \in \vartheta$ in the following of this paper.

Before proceeding further, let us introduce the definition of MDADT.

Definition 1 (*[20]*). For a switching signal $r(k)$ and any $k_2 \ge k_1 \ge 0$, let $N_{rp}(k_2, k_1)$ be the switching numbers that the p_{th} subsystem is activated over the interval $[k_1, k_2)$ and $T_p(k_2, k_1)$ be the total running time of the p_{th} subsystem over the interval $[k_1, k_2]$, $p \in \vartheta$. We say that $r(k)$ has a mode-dependent average dwell time τ_{ap} if there exist positive numbers N_{0p} (we call N_{0p} the mode-dependent chatter bounds here) and τ_{ap} such that

$$N_{rp}(k_2, k_1) \le N_{0p} + \frac{T_p(k_2, k_1)}{\tau_{ap}}, \forall\, k_2 \ge k_1 \ge 0 \tag{2}$$

Remark 1. In Definition 1, for $\forall\, p \in \vartheta$, different value of chatter bound N_{0p} leads different case of switching signals: (i) $N_{0p} = 0$. Condition (2) becomes $N_{rp}(k_2, k_1) \le \frac{T_p(k_2,k_1)}{\tau_{ap}}$ in this case. It thus follows that $N_{rp}(k_2, k_1) \le 0$ if $k_2 \to k_1$, which means that the system is not allowed to switch to mode p. In other words, the switched system will be never running on mode p. (ii) $N_{0p} \ge 1$. It can be inferred from (2) that the system is allowed to jump to mode p for N_{0p} times continuously in some certain time intervals. Therefore, MDADT switching signal with $N_{0p} \ge 1$ can be regarded as a typical one and is considered in this paper.

Remark 2. As for the definition of MDADT, let $N_0 = \sum_{p\in\vartheta} N_{0p}$ and $\tau_p = \tau_q = \tau_a$, $\forall\, p, q \in \vartheta$. It can be computed that the total number of admissible switchings between any time instants k_2 and k_1 as

$$\begin{aligned} N_r(k_2, k_1) &= \sum_{p\in\vartheta} N_r(k_2, k_1) \\ &= \sum_{p\in\vartheta} N_{0p} + \sum_{p\in\vartheta} T_p(k_2, k_1) \\ &= N_0 + \frac{k_2 - k_1}{\tau_a} \end{aligned}$$

This means that under the constraint of $\tau_p = \tau_q = \tau_a$, $\forall\, p, q \in \vartheta$, a MDADT signal meets the requirement of the ADT switching signal with chatter bound $N_0 = \sum_{p\in\vartheta} N_{0p}$ and ADT τ_a.

For a given sequence of switching instant $k_1, k_2, \ldots, k_s, \ldots$, define the following notation which are both mode-dependent and quasi-time-dependent, for $\forall\, r(k) = p \in \vartheta$