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New stability and stabilization conditions for nonlinear systems with time-varying delay based on delay-partitioning approach

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ABSTRACT

This paper focuses on stability analysis and stabilization of nonlinear systems with interval time-varying delay, modeled by Takagi-Sugeno (T-S) fuzzy approach. To achieve more relaxation in the feasibility region, delay-partitioning approach is used for all integral terms in the Lyapunov-Krasovskii functional (LKF). A fuzzy Lyapunov function is proposed instead of non-integral term in LKF, and moreover, some slack matrices variables are offered to enlarge the design space. By doing this, new delay-dependent stability criteria are obtained. During the derivation of stability conditions, Jensen's integral inequality is applied to deal with integral terms. Furthermore, in this paper the problem of controller design via the parallel distributed compensation (PDC) scheme is studied. Stability and stabilization conditions with less conservative are achieved in terms of linear matrix inequality (LMI). Finally, two numerical examples are presented to show the effectiveness of the proposed results.

1. Introduction

Time delay occurs in many industrial systems such as mechanical transmissions, fluid transmissions, metallurgical processes, and networked control systems [1–5]. Time delay is often a source of instability and performance degradation. However, in some cases it may cause stability of the system [6]. Time delay is usually categorized as time constant and time-varying. Moreover, T-S fuzzy model has been extensively used in stability analysis and control design of complicated nonlinear systems during recent decades [4,7]. The fuzzy model employed by Takagi and Sugeno is shown by IF-THEN rules that describes relations between local input-output of a nonlinear system [8]. The overall fuzzy model of the system is achieved by fuzzy blending of these local models in a convex structure. The fuzzy modeling method is based on sector nonlinearity that provides an exact model of nonlinear systems [8]. Therefore, the topic of stability analysis and stabilization of time-varying delay systems in T-S fuzzy modeling framework has been an interesting research field [4]. Stabilization of fuzzy model is studied via PDC approach, which employs multiple linear controller corresponding to the locally linear models with automatic scheduling performed via fuzzy rules [9,10]. Among the newly published technical approaches in this field, the stability analysis of time-varying delay systems based on delay-partitioning approach is very worth-mentioning because it provides less conservative stability conditions compared to previous methods [9].

Many researchers have suggested various techniques for stability and stabilization of T-S fuzzy systems with time delay [11–18]. One of the most effective techniques is delay-partitioning approach [19–29], in which, interval time delay is divided either uniformly or non-uniformly into multiple segments. Then, a different Lyapunov functional is chosen for each segment, which causes less conservative stability conditions [19]. The more number of segments, the less conservativity is achieved [19]. In [20], a stability criterion has been proposed for continuous systems with multiple time-varying delay components by using delay-partitioning approach, however, this approach has been used only for one of integral terms in LKF. An H_∞ control for nonlinear time-varying delay systems has been designed by using delay partitioning approach for integral terms in LKF based on T-S fuzzy modeling [21]. In [22], a new criterion has been established for integral terms, however, that paper has divided interval time delay into non-uniformly segments. A new LKF has been applied in [23] such that in addition to adopting the approach of [22], new terms have been included in LKF to provide more flexibility and relaxation in the stability conditions. In [28,29], approach of [22] has been employed to present less conservative stability criteria for neural networks with interval time-varying delays. Improved results have been obtained by applying reciprocally convex approach in [28].

However, the existing works have ignored plenty of room for the following reasons: delay-partitioning was used in [20] for only one integral term of LKF and the relationship between the

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augmented state vectors have not been fully taken into account. For reducing the conservatism of [23,28,29], the number of segments should be increased, however, by increasing segments, the number of decision variables and LMI conditions increase. Dimensions of LMI conditions increase by increasing the number of segments [23,26]. In [23–27] , the problems of stability analysis have been handled via common quadratic Lyapunov function approach for non-integral term in LKF, which was mainly conservative. Moreover, in these studies controller design has not been considered [23,25–29].

According to the above discussions, the main purpose of this paper is to develop further improved stability and stabilization criteria for nonlinear systems with time-varying delay. To this end, delay-partitioning approach via fuzzy Lyapunov function, slack matrices, and Jensen's inequality is used to convert the problem into LMI. The construction of the proposed approach is summarized as follows: firstly, inspired from the work [30], a vector is defined that plays the main role to represent LKF in a simple framework. LMI conditions in this form can be directly implemented by MATLAB LMI toolbox. Secondly, a new augmented LKF is proposed by partitioning time delay in all integral terms such that the relationships between the augmented state vectors are fully taken into account. Thirdly, the fuzzy Lyapunov function approach is used instead of the quadratic non-integral term in the LKF that plays a crucial role in conservatism reduction. Conservatism can be reduced by decreasing the number of segments in delay-partitioning approach, however, the number of LMIs and decision variables will increase. This issue is solved using fuzzy Lyapunov function. Fourthly, in the process of stability achievement conditions, some slack matrices which provide new degrees of freedom to the LMI conditions are introduced. Fifthly, controllers via the PDC scheme are designed for stabilization and free weighting matrices are taken to convert BMI problem into LMI one. It should be noted that by applying the proposed approach in this paper and recent presented approaches based on delay-partitioning, the computational complexity will grow up for large system matrices.

The rest of this paper is organized as follows: The main problem is formulated in Section 2 and new stability conditions for T-S fuzzy systems with time-varying delay are proposed in Section 3. In Section 4, controller design via the PDC scheme for stabilization is established and two numerical examples are considered to demonstrate the effectiveness of the proposed approach in reducing conservative in Section 5. In Section 6, main conclusions are contained.

1.1. Notations

Throughout this paper, $\mathbb{R}^n$ denotes the n-dimensional Euclidean space, and $\mathbb{R}^{n\times m}$ is the set of $n\times m$ real matrices, the notation $A>(\geq)B$ means that $A-B$ is positive definite (semi-definite), $I(0)$ is the identity (zero) matrix with appropriate dimensions, A^Tdenotes the transpose of A, $\|\bullet\|$ denotes the Euclidean norm in $\mathbb{R}^n$, "*" denotes the elements below the main diagonal of a symmetric block matrix.

2. Problem formulation

A nonlinear system with time-varying delay can be modeled by a fuzzy system with time-varying delay with r rules as follows:

Rule i: IF $\theta_1(t)$ is M_{i1}, $\theta_2(t)$ is M_{i2}, $\cdots$, $\theta_p(t)$ is M_{ip}, Then

$$\begin{cases} \dot{x}(t) = A_i x(t) + A_{di} x(t-\tau(t)) + B_i u(t), & t\geq 0 \\ x(t) = \varphi(t), & t\in[-\tau,0], \end{cases} \tag{1}$$

where $\theta_1(t)$, $\theta_2(t)$, $\cdots$, $\theta_p(t)$ are known premise variables, $M_{il}(i=1,2,\cdots,r,\ l=1,2,\cdots,p)$ are the fuzzy sets, $x(t)\in\mathbb{R}^n$ is the state vector, $\varphi(t)$ is a continuous vector-valued initial function on $[-\tau,0]$. A_i, A_{di} and B_i are known real constant matrices with appropriate dimensions and $\tau(t)$ is a time-varying functional satisfying

$$0\leq \tau(t)\leq \tau \tag{2}$$

$$\dot{\tau}(t)\leq \mu \tag{3}$$

where τ and μ are constants and known.

By a singleton fuzzifier, product inference, and center-average defuzzifier, the fuzzy model in (1) can be represented by:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} h_i(\theta(t))\{A_i x(t) + A_{di} x(t-\tau(t)) + B_i u(t)\}, & t\geq 0 \\ x(t) = \varphi(t), & t\in[-\tau,0], \end{cases} \tag{4}$$

where K_i, the state feedback gain, should be designed. The overall fuzzy controller can be given by:

$$u(t) = \sum_{i=1}^{r} h_i(\theta(t))K_i x(t)$$

where

$$h_i(\theta(t)) = \frac{\prod_{l=1}^{p} M_{il}(\theta_l(t))}{\sum_{i=1}^{r}\prod_{l=1}^{p} M_{il}(\theta_l(t))},\ i=1,2,\cdots,r$$

$$\theta(t) = (\theta_1(t),\cdots,\theta_r(t))$$

$$h_i(\theta(t)) = w_i(\theta(t))/\sum_{i=1}^{r} w_i(\theta(t))$$

$$w_i(\theta(t)) = \prod_{j=1}^{p} M_{ij}(\theta_j(t))\ \textit{for all } t$$

The term $M_{il}(\theta(t))$ is the grade of membership of $\theta_l(t)$ in M_{il}.

$$\begin{cases} \sum_{i=1}^{r} w_i(\theta(t)) > 0 \\ w_i(\theta(t)) \geq 0, i=1,2,\cdots,r \end{cases}$$

and

$$\begin{cases} \sum_{i=1}^{r} h_i(\theta(t)) = 1 \\ h_i(\theta(t)) \geq 0, i=1,2,\cdots,r \end{cases} \quad \textit{for all } t$$

$$\sum_{k=1}^{r} \dot{h}_k(\theta(t)) = 0 \tag{5}$$

Assumption 1. The upper bound of time derivatives of the membership functions is assumed to be known such that $|\dot{h}_k|\leq\beta_k,(k=1,\cdots,r)$, where $\beta_k,(k=1,\cdots,r)$ are real positive constants.

Lemma 1. [31] *(Jensen's inequality) For any constant matrix $W\in R^{n\times n}$, $W=W^T>0$, scalar $\Upsilon>0$, and vector $\dot{x}:[-\Upsilon,0]\to\mathbb{R}^n$ such that the following integration is well-defined, then:*

$$-\Upsilon\int_{-\Upsilon}^{0}\dot{x}^T(t+s)W\dot{x}(t+s)ds \leq -\frac{1}{\Upsilon}z^T(t)\begin{pmatrix} -W & W \\ * & -W \end{pmatrix}z(t)$$

$$z(t) = \begin{pmatrix} x(t) \\ x(t-\Upsilon) \end{pmatrix}$$