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Research article

Constrained tracking control for nonlinear systems

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ABSTRACT

This paper proposes a tracking control strategy for nonlinear systems without needing a prior knowledge of the reference trajectory. The proposed method consists of a set of local controllers with appropriate overlaps in their stability regions and an on-line switching strategy which implements these controllers and uses some augmented intermediate controllers to ensure steering the system states to the desired set points without needing to redesign the controller for each value of set point changes. The proposed approach provides smooth transient responses despite switching among the local controllers. It should be mentioned that the stability regions of the proposed controllers could be estimated off-line for a range of set-point changes. The efficiencies of the proposed algorithm are illustrated via two example simulations.

1. Introduction

Model predictive control (MPC) is one of the well-known control strategies in the process industry due to its capability of optimally controlling multivariable systems with constraints [1,2]. Although a large number of industrial processes are inherently nonlinear, most of the MPC techniques implemented on the real processes are based on linear models. To enhance the performance, a practical control system with large operating regions needs to deal with nonlinearity explicitly. However, a nonlinear MPC could act inadequately because of the computational complexity generally exists in on-line implementation of the controller. As a result, some alternative approaches to consider the nonlinearity in the controller design have been proposed in the literature [3,4].

Robust model predictive control (RMPC) is a convenient approach to consider the modeling error in the controller design. Meanwhile, nonlinear systems could be represented by linear models along with structured or unstructured uncertainties. Thus, a robust control strategy could be implemented instead of a nonlinear one in order to reduce the computational complexity. Therefore, an RMPC strategy could be applied in nonlinear and uncertain cases concurrently. The RMPC techniques mainly are based on minimization of the worst-case objective function incorporating a set of robustness constraints. In this case, [5] introduced a systematic solution for RMPC problem based on linear matrix inequality (LMI) for systems with polytopic and structured feedback uncertainties which could estimate the stability region of the controller. This method was modified next to decrease computational time [6,7], reduce conservativeness [8–12], or simplify the representation of the uncertainties [13–16]. Moreover, [17] proposed a scheduled RMPC algorithm that enlarges the operating region of the controller introduced in [5] efficiently and later on [18] proposed a method that improves its transient response.

Most of the RMPC methods have been formulated for regulation problems that is steering the system to a fixed point. In practice however, it is often required to track a changing set point. When the set point changes, the stabilizing design of the RMPC may not be valid anymore and/or feasibility of the controller may be lost and the controller fails to track the set point. This issue requires redesigning of the RMPC for each value of the changing set point. In order to solve the tracking problem, [19] introduced RMPC based on a dual-model paradigm which guarantees the offset free tracking of set point changes. Ref. [20] proposed a tracking method which guarantees an H_∞ norm bound with an optimized linear quadratic performance. Refs. [21–23] presented tracking RMPC approaches by using the notion of tubes. In [24], tracking scheduled RMPC is proposed for linear time invariant systems such that a set of controllers should be designed off-line based on a predefined target. Ref. [25] introduced a robust tracking MPC based on derivation of an invariant set which provides both a large stabilizable set and the closed loop performance. Ref. [26] proposed a nonlinear MPC method augmented with a disturbance observer (DOB). The DOB estimates internal and external disturbances and then in addition to feedback MPC, a feedforward

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controller is implemented to compensate the disturbances. In [27], an offset-free MPC approach based on prediction accuracy enhancement via a DOB was developed for a general disturbed system and the computational burden of the proposed method was reduced as compared with the existing offset-free MPCs.

Advantages of RMPC such as giving the region of stability and considering input and state constraints in optimization problem make this controller attractive but it has limitations in applications on systems with large operating regions and changing set-points. This paper is devoted to solve the problem by a novel scheduling RMPC scheme. In the off-line part of the proposed method, a set of local controllers is designed that have appropriate overlaps in their stability regions. An on-line part implements a novel switching strategy which uses the designed controllers in off-line part and also some augmented intermediate controllers that are designed on-line. Some features of the suggested method are listed below

– The proposed controller could be applied for constrained nonlinear systems.
– A prior knowledge of the reference trajectory is not required.
– The asymptotic stability of the closed-loop system is guaranteed under set-point changes without needing to change the control scheme for each value of set-points.
– The stability region of the controller for a determined range of set-point changes could be approximated off-line.
– Implementing augmented intermediate controllers in on-line part could prevent the spikes appearing at the moment of switching between adjacent local controllers in the scheduling scheme.

The paper is organized as follows. In Section 2 some mathematical preliminaries are described. Section 3 presents the proposed set point tracking RMPC strategy. In order to illustrate the effectiveness of the proposed method two examples are presented in Section 4. Finally, Section 5 provides the concluding remarks.

2. Mathematical preliminaries

The proposed regulator in [13] and some useful lemmas and remarks which are used in the remainder of the paper are presented in this section. Let consider a nonlinear discrete-time system represented by

$$x(k+1) = f(x(k), u(k)), \tag{1}$$

and subject to

$$|u_j(k)| \le u_{j,\max},\ j = 1, 2, \ldots, m, \tag{2}$$

$$|x_j(k)| \le x_{j,\max},\ j = 1, 2, \ldots, n, \tag{3}$$

where $x(k) \in R^n$ and $u(k) \in R^m$ are the state and input vectors of the system respectively, $f(.,.)$ is a Lipschitz and C^1 function where $f(0,0) = 0$. To design an RMPC based on [13], at first the nonlinear system in (1) is rewritten in uncertain linear representation form

$$x(k+1) = Ax(k) + Bu(k) + \tilde{f}\left(x(k), u(k)\right), \tag{4}$$

where $A = \partial f/\partial x|_{(0,0)}$, $B = \partial f/\partial u|_{(0,0)}$. Since f is a Lipschitz nonlinearity then $\tilde{f}\left(x(k), u(k)\right) = f\left(x(k), u(k)\right) - Ax(k) - Bu(k)$ is bounded as

$$\tilde{f}\left(x(k), u(k)\right)^T \tilde{f}\left(x(k), u(k)\right) \le \left[x^T(k)u^T(k)\right]W^T W\left[x^T(k)u^T(k)\right]^T, \tag{5}$$

where $W^T W$ is a positive definite matrix and represents the Lipschitz coefficient.

Assumption 1. The pair (A, B) is stabilizable by state feedback control law.

The objective function in MPC which is minimized to optimize performance of the closed-loop system is defined as follows

$$J(k) = \sum_{i=0}^{\infty} \left\{ x(k+i|k)^T Q x(k+i|k) + u(k+i|k)^T R u(k+i|k) \right\}, \tag{6}$$

where $Q > 0$, $R > 0$, $x(k+i|k)$ is state at time $k+i$ predicted based on the measurements at time k and $u(k+i|k)$ is control move at time $k+i$ computed by minimizing $J(k)$ at time k. To solve the given optimization problem for the nonlinear system in (1) via LMI, first one should replace equality in (6) by an inequality which is done by defining an upper bound for $J(k)$. Consider a quadratic function $V(x) = x^T P x$ with $P > 0$ and $V(0) = 0$ satisfies the following inequality at sampling time k

$$V(x(k+i+1|k)) - V(x(k+i|k)) \le -x(k+i|k)^T Q x(k+i|k) - u(k+i|k)^T R u(k+i|k). \tag{7}$$

By summing both sides of (7) from $i = 0$ to $i = \infty$ one could find that

$$x(\infty|k)^T P x(\infty|k) - x(k|k)^T P x(k|k) \le -J(k). \tag{8}$$

For the asymptotic stability of the closed-loop system, $x(\infty|k)$ must be zero and thus to have an asymptotic stability it follows that

$$J(k) \le V\left(x(k|k)\right) \le \gamma, \tag{9}$$

where γ is a positive scalar that can be an upper bound for (6). As a result, the RMPC problem is defined as follows

Theorem 1. *Consider system (1) subject to input and state constraints as (2) and (3). Let $x(k|k)$ be the measured state $x(k)$ at sample time k. Then, the state feedback matrix $F(k)$ in the control law $u(k+i|k) = F(k)x(k+i|k)$ that minimizes the upper bound $V(x(k|k))$ of the objective function $J(k)$ at instant k is given by $F = YX^{-1}$, where $X > 0$ and Y are obtained from the solution of the following optimization problem with variables γ, ξ, X, Y, M, N, $Z = [X;Y]$.*

$$\min_{\gamma,\xi,X,Y,M,N} \gamma \tag{10}$$

subject to

$$\begin{bmatrix} I & x(k|k)^T \\ x(k|k) & X \end{bmatrix} \ge 0,\ X > 0, \tag{11}$$

$$\begin{bmatrix} X & \sqrt{1+\varepsilon}(AX+BY)^T & \sqrt{1+1/\varepsilon}(WZ)^T & (Q^{1/2}X)^T & (R^{1/2}Y)^T \\ \sqrt{1+\varepsilon}(AX+BY) & X & 0 & 0 & 0 \\ \sqrt{1+1/\varepsilon}(WZ) & 0 & \xi I & 0 & 0 \\ Q^{1/2}X & 0 & 0 & \gamma I & 0 \\ R^{1/2}Y & 0 & 0 & 0 & \gamma I \end{bmatrix} \ge 0, \tag{12}$$

$$X - \xi I \ge 0, \tag{13}$$

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