ISA Transactions ∎ (∎∎∎∎) ∎∎∎–∎∎∎

Contents lists available at ScienceDirect

ISA Transactions

journal homepage: www.elsevier.com/locate/isatrans

Research article

Further triple integral approach to mixed-delay-dependent stability of time-delay neutral systems

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ARTICLE INFO

Article history:
Received 21 August 2016
Received in revised form
4 April 2017
Accepted 18 May 2017

Keywords:
Neutral systems
Asymptotical stability
Triple integral Lyapunov functional
Mixed-delay-dependence
Time-varying delay

ABSTRACT

This paper studies the asymptotic stability for a class of neutral systems with mixed time-varying delays. Through utilizing some Wirtinger-based integral inequalities and extending the convex combination technique, the upper bound on derivative of Lyapunov-Krasovskii (L-K) functional can be estimated more tightly and three mixed-delay-dependent criteria are proposed in terms of linear matrix inequalities (LMIs), in which the nonlinearity and parameter uncertainties are also involved, respectively. Different from those existent works, based on the interconnected relationship between neutral delay and state one, some novel triple integral functional terms are constructed and the conservatism can be effectively reduced. Finally, two numerical examples are given to show the benefits of the proposed criteria.

1. Introduction

The stability of time-delay systems has become a focused topic of theoretical and practical importance since time-delays are always existent in many fields, such as physics, economy, and industrial systems. In a large number of control systems, it is well known that the presence of time-delay often leads to the oscillation, instability, or other poor performances. Therefore, the research on stability in many kinds of time-delay systems has received extensive attention [1–44]. Meanwhile, neutral functional differential equations can be used to describe the mathematical models of some control systems, in which the time-delays simultaneously exist in both state and its state derivative. This kind of model is called as time-delay neutral system and it can be often encountered in various fields, such as population change, heat exchanging, and steam process. Hence, many sufficient conditions have been proposed for the stability in a variety of time-delay neutral systems [14–44].

In [1–6,14,15], through choosing special L-K functionals and using LMI approach, the asymptotical stability for neutral systems with constant delays has been studied. In [16,17], some uneasy-to-test conditions on exponential stability for variable neutral ones have been derived, in which time-varying delay was studied. Yet, in practical cases, owing to that accurate mathematical model cannot be easily achieved, many present works always assume that the system parameters exist norm-bounded uncertainties [1,3,5,6,18–22] and the system simultaneously contains both linear part and nonlinear one [23–29]. As for stability issue, on one hand, in [18–22,30–32,41,42], with the help of several effective techniques, some LMI-based stability criteria on uncertain ones have been presented. On the other hand, in [23–29], through treating the restricted nonlinearity as system disturbance, some delay-dependent criteria have also been presented in terms of LMIs. Meanwhile, since the discrete delay can be introduced into communication channels since it is ubiquitous in signal transmission, a system usually has a special nature due to the presence of an amount of parallel pathways with a variety of axon sizes and lengths. Such an inherent nature can be suitably modeled as so-called distributed delay. Therefore, some researchers have studied the robust stability of neutral systems with distributed delays [18,21,22,26–29]. Together with some improved delay-partitioning ideas, the works [34,35] have established the stability criteria and their conservatism can be greatly reduced by thinning the delay intervals. There also exist some works involving the effect of other factors, such as stochastic disturbance [31,32,38], leakage delays with impulse [33], Markov jumping [37], H_∞ performance [39], neutral Lur'e type [40–42], and their applications to descriptor/singular systems [43,44]. It is worth pointing out that, since the triple Lyapunov approach was put forward in [7], it has received a great deal of improvements and been employed to study the global stability for time-delay neutral systems [8,10–13,34,36].

It is worth noting that, based on some existent works, when the L-K stability theory was used to address the issue on

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http://dx.doi.org/10.1016/j.isatra.2017.05.010
0019-0578/

Please cite this article as: Wang T, et al. Further triple integral approach to mixed-delay-dependent stability of time-delay neutral systems. ISA Transactions (2017), http://dx.doi.org/10.1016/j.isatra.2017.05.010

delay-dependence, many effective methods have been proposed, such as free-weighting matrices, various integral inequalities, convex combination technique, delay-partitioning idea, multiple Lyapunov functional, and so on [23–44]. Though the results above were elegant, two following points are still existent waiting for the improvements. ***Firstly***, when the works above estimated some integral terms in the L-K functional's derivative, such as $\int_{t-\tau}^{t}\dot{x}^T(s)P\dot{x}(s)ds$ and $\int_{t-\tau}^{t}\int_{t-\tau}^{s}\dot{x}^T(u)Q\dot{x}(u)duds$, much useful information still has been neglected. Most recently, the works [15–18] have presented some novel Wirtinger-based inequalities to overcome the shortage in some degree. However, no matter how tightly they gave the upper estimations on these integral terms, the involved time-delays were still constant. Yet, in practice, time-delays have to be always variable in certain intervals. ***Secondly***, as is well known, there always exist two kinds of time-delays in neutral systems. One is state delay and the other is included in state's derivative. Based on many practical models, such two kinds of time-delays are always different [24–28]. However, most existent works always employed the information on each delay individually to choose the L-K functional and derive stability results [27–40]. Yet during the analysis, it will be of much more significance to consider the interconnected relationship between them. Though the works [15,18] have given some preliminary discussions on this point, the involved delays were constant and the Lyapunov functional forms seemed to be simple, which leave much room for the further research. Meanwhile, since the triple Lyapunov technique was put forward in [7], it has received much attention during tackling the time-delay systems [14–19,40,42]. However, as for time-delay neutral systems, few works have employed such technique to study the interconnected relationship between neutral delay and the state one. Overall, some efficient approaches need to be put forward to derive more elegant results.

Motivated by the discussion above, in this paper, the global stability for a class of time-delay neutral systems will be deeply studied. Together with the interconnection between two kinds of time-delays, an improved Lyapunov-Krasovskii functional will be constructed and some novel integral inequalities will be utilized to give much tighter upper bound on L-K functional's derivative. The derived criteria are presented in terms of LMIs and can be easily tested, in which the nonlinearity and parameter uncertainties will be also considered. Finally, two numerical examples will be presented to illustrate the reduced conservatism of the derived results.

Notations: The term L-K functional is the abbreviation of Lyapunov-Krasovskii functional; the set $\mathbf{R}^n$ denotes the n-dimensional Euclidean space and $\mathbf{R}^{n\times m}$ is the set of $n\times m$ real matrices; I denotes an identity matrix of appropriate dimension; $\mathbf{sym}\{X\}$ means $\mathbf{sym}\{X\}=X+X^T$ and $\|X\|$ stands for Euclidean norm of X; and the symmetric term in a symmetric matrix is denoted by $*$, i.e., $\begin{bmatrix} X & Y \\ Y^T & Z\end{bmatrix}=\begin{bmatrix} X & Y \\ * & Z\end{bmatrix}$.

2. Problem formulations and lemmas

In this paper, we consider the following neutral systems with time-varying delays as

$$\dot{x}(t)-C\dot{x}(t-\tau_1(t))=Ax(t)+Bx(t-\tau_2(t))+Df\big(t,x(t),\ \dot{x}(t-\tau_1(t)),x(t-\tau_2(t))\big),\ t\ge t_0;$$
$$x(t)=\phi(t),\ t\le t_0, \tag{1}$$

where $x(t)\in\mathbf{R}^n$ is the state of system (1) and A, B, C, D are constant matrices of appropriate dimensions with $\|C\|<1$.

The following assumptions on the system (1) will be utilized throughout this paper.

H1. The functions $\tau_i(t)(i=1,2)$ denote the time-varying delays satisfying

$$0\le\tau_i(t)\le\hat{\tau}_i,\quad \upsilon_i\le\dot{\tau}_i(t)\le\mu_i(i=1,2). \tag{2}$$

Here we denote $\bar{\tau}_i(t)=\hat{\tau}_i-\tau_i(t)$, $\bar{\mu}_i=\mu_i-\upsilon_i(i=1,2)$, $\tau_{21}=\hat{\tau}_2-\hat{\tau}_1$, and $\delta_{21}=\hat{\tau}_2^2-\hat{\tau}_1^2$.

H2. There exist constant matrices $\Xi_i(i=1,2,3)$ such that the nonlinear function $f(t,\cdot,\cdot,\cdot)$ satisfies

$$\|f(t,a,b,c)\|\le\|\Xi_1a\|+\|\Xi_2b\|+\|\Xi_3c\| \tag{3}$$

with the vectors $a,b,c\in\mathbf{R}^n$.

Remark 1. In H1, on one hand, when time-delays $\tau_i(t)(i=1,2)$ are constant ones, one can easily check that $\upsilon_i=\mu_i=0(i=1,2)$; on the other hand, when time-delays $\tau_i(t)(i=1,2)$ are time variable, it is easy to obtain that the values of $\upsilon_i(i=1,2)$ have to be less than 0 and the ones of $\mu_i(i=1,2)$ have to be greater than 0, which guarantees $\tau_i(t)(i=1,2)$ to be bounded in the intervals $[0,\hat{\tau}_i](i=1,2)$. Yet many present works aimed to study the upper bound of $\dot{\tau}_i(t)(i=1,2)$ but neglected the information on its lower bound, which would unavoidably result in some conservatism.

In what follows, some lemmas will be given for the proof procedure.

Lemma 1 (*[10]*). *Let $\varphi(\cdot)$ be a differential function: $[a,b]\to\mathbf{R}^n$. For the positive matrices $M\in\mathbf{R}^{n\times n}$, and $N_1,N_2,N_3\in\mathbf{R}^{4n\times n}$, the following inequality holds:*

$$-\int_a^b\dot{\varphi}^T(s)M\dot{\varphi}(s)ds\le\chi^T\Omega\chi,$$

where

$$\begin{aligned}\Omega &= \mathbf{sym}\{N_1\Theta_1+N_2\Theta_2+N_3\Theta_3\}+(b-a)\\ &\quad\left(N_1M^{-1}N_1^T+\frac{1}{3}N_2M^{-1}N_2^T+\frac{1}{5}N_3M^{-1}N_3^T\right),\\ e_i &= \left[0_{n\times(i-1)n}I_n0_{n\times(4-i)n}\right](i=1,2,3,4);\\ \Theta_1 &= e_1-e_2;\ \Theta_2=e_1+e_2-e_3;\ \Theta_3=e_1-e_2-6e_3+6e_4;\\ \chi &= \left[\varphi^T(b)\varphi^T(a)\frac{1}{b-a}\int_a^b\varphi^T(s)ds\frac{2}{(b-a)^2}\int_a^b\int_a^s\varphi^T(u)duds\right]^T.\end{aligned}$$

Lemma 2 (*[11]*). *For an any constant matrix $M>0$, the following inequality holds for all continuously differentiable function $\varphi(\cdot)$ in $[a,b]\to\mathbf{R}^n$:*

$$-(b-a)\int_a^b\varphi^T(s)M\varphi(s)ds\le-\left(\int_a^b\varphi(s)ds\right)^TM\left(\int_a^b\varphi(s)ds\right)-3\Theta^TM\Theta,$$

where $\Theta=\int_a^b\varphi(s)ds-\frac{2}{b-a}\int_a^b\int_a^s\varphi(u)duds$.

Lemma 3 (*[11]*). *For an any constant matrix $M>0$, the following inequality is true for all continuously differentiable function $\varphi(\cdot)$ in $[a,b]\to\mathbf{R}^n$:*

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