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Research article

An improved stopping condition guarantee recovery of sparse signal via Subspace Pursuit method

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The Subspace Pursuit (SP) algorithm is one of greedy pursuit methods which is used to reconstruct of K-sparse signal. Unlike existing condition produced by Dai and Milenkovic in 2004 that suggests the residual value of current iteration is reduced from the previous iteration, our approach eliminates useless information by reducing the number of iterations used to detect the correct support set. This operation is done by suggesting a new halting condition that can capture the best support set which can give the best representation of the reconstructed signal. The new halting conditions enhanced the SP algorithm to low computational complexity and reconstruction accuracy of the sparse signal.

A mathematically proven for two halt condition: noiseless setting, and noisy setting for signal affected by Gaussian noise. An error bound relation also is driven.

In this paper, we try also to relax the restricted isometry constant RIC value to narrows the gap between the known bounds and ultimate performance, which it produced by Dai.

Simulation results show that the new halting condition can overpass best results produce by earlier iteration and rise time consume. Our new halting condition can catch this earlier iteration and enhanced SP algorithm results.

1. Introduction

The motivation of this paper is to improve the SP algorithm by modifying the halting condition suggested by Dia. In each iteration, this new condition tries to keep track of an estimated support consisting of K elements (non-zero sparse input) by removing and adding elect elements from and to the candidate set, while still ensuring that the overall computational complexity remains competitive with old SP. Reducing the number of iteration process leads to reduce the time needed to collect the necessary data and these play a very important role in some areas, especially in the medical field when using the MRI device, where gathering important information in slight time is the aspirations of the modern science.

In 2004 E. J. Candes, T. Tao, and David L. Donoho has introduced the Compressed Sensing (CS) theory, since then it becomes an increasingly fast emerging research field [1]. The challenge in CS is to reconstruct this sparse signal from few measurements as possible as it could.

CS attracted much interest in the research community and found wide-ranging applications such as in astronomy [2], communications [3], image and video processing [4], biology [5], medicine [6,7], and radar [8].

The term "single pixel camera" [6]**:** that first developed at Rice University was the best known device for compressive sensing. Compressive Sensing Microarray (CSM) is one of microarray sequencing method that is considered as new device used for DNA identification of organisms [5].

CS differs from classical sampling in two important aspects; first it differs when the signal at a specific point in time, and CS acquires measurement by using inner products between the signal and more general test function. The randomness always plays a role in design models of the CS [9].Secondly, the big deal here is how to recover the original signal from the compression measurements. The standard CS theorem is based on a sparse signal model and uses an undetermined system of linear equations [10].

Many signals are usually sparse or compressible on some basis such as images or audio signal. Even though the signal is sparse, it is insignificant action to reconstruct original signals from compacted readings, as long as we couldn't know the location of the non-zero coefficient of that vector.

For a large sparse signal hopefully, it is recovered correctly and precisely by fewer numbers of measurements as much as possible.

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In spite of the fact that this appears to be conceivable in theory, the difficulty is in assembling calculations to performing the precision and efficiency of reconstructing.

Several low complexity reconstruction methods are used nowadays as a recovery method. Some of these include Convex Optimization: like Basis Pursuit (BP) and Basis Pursuit De-Noising (BPDN), Iterative Greedy Algorithms like Matching Pursuit (MP), Orthogonal Matching Pursuit (OMP) [11], the Regularized OMP (ROMP), and Compressive Sampling Matching Pursuit (CoSaMP) [12].

The simple idea behind the use of a greedy method is to find the support for an unknown signal sequentially. The support set contains indices of non-zero elements of a sparse vector.

In [10] and [13] Wei Dai and Milenkovic proposed a new greedy algorithm for recovering a sparse signal, this method called subspace pursuit. The RIP for SP has been investigated in [13]. This algorithm allowing for exact and approximate signal reconstructed, and can work for noiseless and noisy case respectively.

SP algorithm combines a simple method for re-evaluating the reliability of all candidates at each iteration of the process.

The basic idea behind the SP method depends upon the A^{*} order statistic algorithm.

Dai adopt in his suggested algorithm a stopping criterion $\left\|r^{j}\right\| \geq \left\|r^{j-1}\right\|$ which describe that when the previous residual is less than the current residual, the iteration is halted and the current support is equal to original support set [10].

An obvious drawback of SP is that there is no guarantee of the overall reconstruction quality of the support sets. In particular, since the SP method depend on one condition for terminate the iteration, there is no guarantee that the residual error due to r^{j} is lower than that due to r^{j-1}.

The central issue facing the decoding operation for the signal recovering performance, is choosing the right step that give a reasonable measurements produce from the product of an orthogonal projection matrix and the random sensing matrix, and this can be done by denoting the right halting condition that produces a recovering support set which has a converges value compare to original one. Based on the developed analysis in this paper, we derive a more accurate stopping condition as it compared to the one produce by Dai and Milenkovic [10,13].

By using our proposed halting conditions, we can guarantee perfect and stable signal recovering. The requirement on the RIC of the sensing matrix Φ can be relaxed to $\delta_{K+1} < \frac{\sqrt{K+1}+\sqrt{K}}{2K+1}$. Specifically, assuming that the sensing matrix Φ satisfies RIP of order $3K$, we can show that δ_{3K} guarantee exact and stable support identification via SP in the noiseless case (and approximately in noisy case). Once the correct support set is determined, the non-zero coefficients are calculated using pseudoinversion process. Our target, thus improves the results in [10] and [13].

A lot of theories were suggested in this field depend on SP stopping condition, so the major challenge we faced here is to find a convincing mathematical expression which is superior to the previous one, and provide accurate and stable performance with existing and absence of noise.

The limitation of our new suggestion halting condition is the same drawback as the old condition of ordinary SP method, which is, if the measurement rate is very low, the performance is not overcome the old greedy methods such as CoSaMP method, the SP method showing it's best performance when the measurement rate reaching somewhat a good value. Actually we cannot consider this as drawback because even the old greedy methods show better reading, but it's still unacceptable results. This feature can be observed in detail at the simulation results section.

To declare the standard CS problem, which achieves a signal $x \in \mathbb{R}^{N}$, which has a $K-sparse$ input; the linear measurements

$$y = \Phi x$$

Where $\Phi \in \mathbb{R}^{M\times N}$ represents a random measurement (sensing) matrix, and $y \in \mathbb{R}^{M}$ represents the compressed measurement signal [14].

Iterative greedy depends on its search method upon an estimation of the implicit support set of a sparse vector [15].

A new compressed sensing noiseless and noisy signal reconstruction based on derived new stopping condition based on the greedy algorithm is introduced in this paper. The new proposed theory has less computational complexity and faster than the old SP greedy algorithm.

The sensing matrix Φ is said to satisfy RIP of order K [16] if $0 < \delta_{K} < 1$ such that

$$\left(1-\delta_{K}\right)\|x\|_2^2 \leq \|\Phi x\|_2^2 \leq (1+\delta_{K})\|x\|_2^2$$

Which it hold for all $K-sparseX$. The value δ_{K} is called restricted isometry constant (RIC) of the sensing matrix Φ.

If the measurements corrupted by some noise (n), then the measurement vector y can be represented as

$$y = \Phi x + n$$

where $n \in \mathbb{R}^{M}$ represents the additive white Gaussian noise (AWGN) which have zero mean and variance σ^{2} [17].

In the presence of noise measurement, different algorithms have been developed to approximate the original signal, suppose some signal parameters and certain noise are known [18].

The RIP for SP has been investigated by Wei Dai and Milenkovic in [13]. By studying the proposed approach of [10,13], we show in this paper that to guarantee stable signal reconstruction via the SP method, a new stopping condition can be achieved and the requirement of the RIC for sensing matrix can be relaxed further. Our bound improves the results in [10]: for the absence of noise, the RIC value was $\delta_{3K}<0.165$; and for noisy case $\delta_{3K}<0.083$.

1.1. Notation

We use $\Phi \in \mathbb{R}^{m\times N}$ to denote the sensing matrix where $<<N$. For $x \in \mathbb{R}^{N}$ is the original input sparse signal having a length equal to N, and $x_{T} \in \mathbb{R}^{|T|}$ denotes the vector whose entries consists of those of x indexed by T. In this paper K denote sparse vector, $y \in \mathbb{R}^{M}$ is the measurement vector, Φ_{T} denotes submatrix of Φ with the element columns chosen from the ordered set T. Also T^{k} denote the support set of the current iteration while T^{k-1} denote the support sets which had been calculated in the previous iteration, y_{r}^{k} is the residual of the current iteration, y_{p} is the projection residue, and T_1â§¹T_2 is represented the element set found in T_1 but not in T_2 . We use $|I|$ to denote the cardinality set, and $()^{*}$ stands for the transpose operation, $\|X\|$ represent the ℓ_2-norm , $\Phi^{\dagger}$ is the moore-penrose pseudo inverse of matrix Φ. We write $\tilde{x}_{s} \in \mathbb{R}^{N}$ to denote the zero padded for x_{s}. I is the identity matrix of a proper dimension.

2. New RIC value vs old value

Mo and Shen [19], suppose the upper bound on δ_{K+1} can further relax to $\delta_{K+1} < \frac{1}{\sqrt{K}+1}$. Ling-Hua and Jwo-Yuh [17] relax the upper bound to $\frac{\sqrt{4K+1}-1}{2K}$ trying to narrows the gap between known bound and conjecture made by Wang and Shim [20] which is equal to $\delta_{K+1} < \frac{1}{\sqrt{K}}$

This paper shows that the upper bound on δ_{K+1} in the reconstruction condition can be improved

$$\delta_{K+1} < \frac{\sqrt{K+1}+\sqrt{K}}{2K+1} \tag{1}$$

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