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Research article

A novel body frame based approach to aerospacecraft attitude tracking ☆

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ABSTRACT

In the common practice of designing an attitude tracker for an aerospacecraft, one transforms the Newton-Euler rotation equations to obtain the dynamic equations of some chosen inertial frame based attitude metrics, such as Euler angles and unit quaternions. A Lyapunov approach is then used to design a controller which ensures asymptotic convergence of the attitude to the desired orientation. Although this design methodology is pretty standard, it usually involves singularity-prone coordinate transformations which complicates the analysis process and controller design. A new, singularity free error feedback method is proposed in the paper to provide simple and intuitive stability analysis and controller synthesis. This new body frame based method utilizes the concept of Euleraxis and angles to generate the smallest error angles from a body frame perspective, without coordinate transformations. Global tracking convergence is illustrated with the use of a feedback linearizing PD tracker, a sliding mode controller, and a model reference adaptive controller. Experimental results are also obtained on a quadrotor platform with unknown system parameters and disturbances, using a boundary layer approximated sliding mode controller, a PIDD controller, and a unit sliding mode controller. Significant tracking quality is attained.

1. Introduction

As common in many robotics control problems, the orientation of an aerospacecraft with respect to a fixed inertial frame has to be properly represented before its dynamic equations can be written, upon which control laws can be designed for position and attitude tracking. An aerospacecraft's position is usually described using a Cartesian coordinate system (*XYZ* axes) or a Geographic Coordinate System (GCS, consisting of latitude, longitude, and altitude), whilst the orientation is commonly expressed in Euler angles, Direction Cosine Matrices (DCM), and unit Quaternions, each with different advantages and disadvantages. In the common practice of designing an attitude tracker for an aerospacecraft, one transforms the Newton-Euler rotation equations to obtain the dynamic equations of the chosen attitude metrics, upon which the controller is designed and analysed. Although this design methodology is pretty standard, it usually involves singularity-prone coordinate transformations which complicates the analysis process and controller design. A new, singularity free error feedback variable is proposed in the paper to provide simple and intuitive stability analysis and controller synthesis. Some of the existing attitude representations and design methods are explained below, which then lead to this paper's motivation.

The Euler angle representation [1–6] is the most adopted and intuitive model. By looking at the Euler angles directly, one can easily visualize the orientation of an aerospacecraft following three successive rotations about its axes (order *zyx* in the aerospace convention). However, due to its three-angle representation nature, the gimbal lock problem [7] appears when the second axis of rotation becomes parallel to the first or the third, during which one degree of freedom is lost and the Euler angle rate matrix becomes singular.

To avoid singularity, the DCM [8–10] or unit quaternions [11–13] can be used, utilizing nine and four numbers to represent the orientation, respectively. A DCM is an orthonormal rotational matrix, whilst a unit quaternion is a unit-length hypercomplex number. Visualization of an aerospacecraft orientation using these two expressions are not as easy as using Euler angles.

The Euleraxis-angle representation (sometimes known as the Eigenaxis-angle, with its variants including Rodrigues parameters, Modified Rodrigues parameters, and Gibbs vector) [14–17] describes orientation by a fixed axis in the inertial frame and an angle of rotation. Although compact, this representation has seen fewer applications as the derivation of its kinematics equations can be rather tedious and problematic.

A general procedure can be summarized for deriving the attitude dynamic equations. First, choose the desired orientation representation method. Second, transform the Newton-Euler rotation equations so that the dynamic equations of the chosen attitude metrics are obtained. Third, design controllers that utilizes the feedback the complete or part of the attitude metrics and their

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Nomenclature

Key notations

$\lvert\cdot\rvert$	absolute value
$\lVert\cdot\rVert_1$	vector 1-norm
$\lVert\cdot\rVert$	vector 2-norm
$\sup(\cdot)$	supremum
$\rho(\cdot)$	spectral radius
diag $(\cdot)$	diagonal matrix of vector
$I_{n\times n}$	identity matrix of order n
I	inertia tensor
$\boldsymbol{M}$	body frame applied torque vector
$\boldsymbol{\omega}$	body frame angular rate vector
$[\boldsymbol{\omega}\times]$	cross product matrix of $\boldsymbol{\omega}$
$\boldsymbol{q}_m$	quaternion m
$\otimes$	quaternion multiplication
$\boldsymbol{s}$	sliding surface vector
$\boldsymbol{\beta}_e$	body error angle vector
x_d	desired value of x
$\hat{x}$	estimated value of x
$\tilde{x}$	tracking/estimation error of x
sgn(x)	sign of x; equals zero when x is zero
sat(x)	equals sgn(x) for $\lvert x\rvert \geq 1$; equals x for $\lvert x\rvert < 1$
$\succ$	component-wise larger than
$\prec$	component-wise less than

time derivatives to control such a highly nonlinear plant.

From a mathematical perspective, the above procedure is standard and has proven to be reliable with many variations, including quaternion feedback [14,18,19], geometric tracking with rotation matrices [9]. However, due to the high nonlinearity of the system, which is especially amplified when the dynamics are expressed in terms of the attitude metrics and their rates with respect to the inertial frame using different orientation expressions, analyses and control designs are rendered cumbersome. From a practical point of view, however, all the aerospacecraft needs to know is how far to turn about the axes of its own frame in order to reach the desired orientation. If the dynamic equations are expressed completely in the body frame of the aerospacecraft, controller designs and system analyses can be greatly simplified. This motivates the novel body frame modelling and design methods presented in this paper, which can be seen as an extension and a new interpretation of the error Euleraxis-angle as observed in the body frame. Using the proposed methods, global attitude tracking performance is illustrated with the use of a feedback linearizing PD tracker, a sliding mode controller, and a model reference adaptive controller in simulations. Experiments were also completed on a quadrotor using a sliding mode controller, a PIDD controller, and a unit sliding mode controller.

The rest of the article is structured as follows: first, the working principles of Euler angles, DCM, unit quaternions, and Euleraxis-angle representations are revisited in Section 2, along with the discussion of a standardized inertial frame based controller synthesis and stability analysis method. Second, the novel body frame based controller synthesis and stability analysis methods are derived and presented in Section 3. Third, simulations and experimental results are presented in Sections 4 and 5.

2. Frames and rotation models

The right-hand frame and the Cartesian coordinate system will be used throughout this paper, assuming a flat, stationary Earth. An inertial frame is fixed to the stationary observer on Earth with X, Y, Z axes pointing north, east, and towards the center of the Earth, respectively. A body frame is attached to the center of gravity of the aerospacecraft with the x, y, z axes pointing at the nose, to the right, and to the belly of the vehicle. Different orientation representations can then be used to describe the orientation of the body frame with respect to the inertial frame, with a common set of body frame Newton-Euler rotation equations,

$$\boldsymbol{M} = I\dot{\boldsymbol{\omega}} + [\boldsymbol{\omega}\times]I\boldsymbol{\omega}, \tag{1}$$

where $\boldsymbol{M} = [M_x, M_y, M_z]^T$ is the vector of body frame applied torques, $\boldsymbol{\omega} = [p, q, r]^T$ is the vector of body frame rotation rates, $I \in \mathbb{R}^{3\times 3} > 0$ is the symmetric positive definite aerospacecraft moment of inertia tensor, and $[\boldsymbol{\omega}\times]$ is the skew-symmetric cross product matrix of $\boldsymbol{\omega}$ such that

$$I = \begin{bmatrix} a_1 & a_2 & a_3 \\ * & a_4 & a_5 \\ * & * & a_6 \end{bmatrix}, \quad [\boldsymbol{\omega}\times] = \begin{bmatrix} 0 & -r & q \\ r & 0 & -p \\ -q & p & 0 \end{bmatrix}. \tag{2}$$

For later convenience, we denote $\boldsymbol{a} = [a_1, a_2, \ldots, a_6]^T$. Note that (1) describes the inertial and gyroscopic properties of the rotating aerospacecraft, upon which the inertial frame attitude dynamic equations can be derived with increased sophistication to include more physical effects, such as aerodynamics.

2.1. Euler angles

Euler angles generalize the description of orientation using successive rotations from one frame to another, with twelve selectable rotation orders [20]. In the aerospace convention, Euler angles are defined using the zyx right-hand rotation order, with the respective angles of rotation denoted as yaw (ψ), pitch (θ), and roll (ϕ). With $\boldsymbol{\Theta} = \left[\phi, \theta, \psi\right]^T$, the Euler angle dynamic equations can be derived as

$$\dot{\boldsymbol{\Theta}} = J^{-1}\boldsymbol{\omega} = \begin{bmatrix} 1 & \mathrm{s}\phi\mathrm{t}\theta & \mathrm{c}\phi\mathrm{t}\theta \\ 0 & \mathrm{c}\phi & -\mathrm{s}\phi \\ 0 & \mathrm{s}\phi(\mathrm{c}\theta)^{-1} & \mathrm{c}\phi(\mathrm{c}\theta)^{-1} \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix}, \tag{3}$$

in which c, s, t stands for cosine, sine, and tangent functions. Notice that the Euler angle rate matrix J^{-1} becomes singular when the $\theta = \pm\frac{\pi}{2}$, which can be avoided by switching the rotation order when θ gets close to the critical values.

2.2. Direction cosine matrices

DCMs are orthonormal rotation matrices that rotate a vector from one frame to another, and can be constructed using Euler angles. The three rotations in the Euler angle representation correspond to the three rotational matrices R_z, R_y, and R_x, which in successive rotations would give

$$R_{xyz} = R_x R_y R_z = \begin{bmatrix} \mathrm{c}\theta\mathrm{c}\psi & \mathrm{c}\theta\mathrm{s}\psi & -\mathrm{s}\theta \\ \mathrm{c}\psi\mathrm{s}\phi\mathrm{s}\theta - \mathrm{c}\phi\mathrm{s}\psi & \mathrm{c}\phi\mathrm{c}\psi + \mathrm{s}\phi\mathrm{s}\theta\mathrm{s}\psi & \mathrm{c}\theta\mathrm{s}\phi \\ \mathrm{s}\phi\mathrm{s}\psi + \mathrm{c}\phi\mathrm{c}\psi\mathrm{s}\theta & \mathrm{c}\phi\mathrm{s}\theta\mathrm{s}\psi - \mathrm{c}\psi\mathrm{s}\phi & \mathrm{c}\phi\mathrm{c}\theta \end{bmatrix}. \tag{4}$$

By differentiating (4) with respect to time, it can be found that [20]