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Research article

Design of multiloop PI controllers based on quadratic optimal approach

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ABSTRACT

The LQR methodology is extended to produce multiloop PI controllers in this study. With this method, a multivariable P controller is first obtained and then diagonalized using an iterative procedure. The resulting controller is further complemented with the integral action in a similar process. The proposed tuning process explores different values of the control weight matrix to balance the output error and control signal in both stages, and is refined through additional indices related with the error to reference tracking, disturbance rejection and the associated control use. In addition, the diagonalization procedure is generalized to obtain multiloop versions of existing multivariable PI controllers. The developed theory was applied to the design of multiloop controllers for a distillation column as well as the diagonalization of existing controllers for a nonlinear CSTR. The tuning procedure allowed the synthesis of multiloop PI controllers with performance indices comparable to those reported by other authors. Furthermore, diagonalized controllers exhibited a similar operation as the original multivariable ones.

1. Introduction

Regardless of the several available strategies for process control, proportional-integral (PI) and proportional-integral-derivative (PID) algorithms are still the preferred control methods for most modern industrial processes due to their simple structure and effectiveness [1,2]. Moreover, it is widely accepted that PI and PID settings can be designed to be robust, which is a desirable property for both safety and to avoid process failures against uncertainties [3]. Nevertheless, the design of PI/PID controllers simultaneously achieving a correct control system operation (adequate set-point tracking, disturbance rejection and insensibility to model uncertainty) is not an easy task [4,5], especially in processes having multi-input–multi-output (MIMO) dynamics with complex interactions among the variables [6]. Therefore, the development of new tuning strategies for PI/PID controllers in MIMO processes remains an active topic, as demonstrated by several studies focusing on different strategies such as dynamic set-point weighting [7], relative normalized gain array [8], pole placement [9], internal model control [1], gain and phase margin specifications [6], steady-state gain matrix[5] and multiobjective optimization [2,3,10,11].

The quadratic optimal control is a state-space design methodology whereby quadratic performance indices involving the control signal and the state variables are minimized to obtain a multivariate feedback controller known as linear quadratic regulator (LQR). Recent studies using LQR approach have focused on finding an effective choice of the weighting matrices regulating the penalties on the deviation in the trajectories of the state variables and control signal [12], the introduction of pole placement specifications [13] and its application to fractional order systems [14]. In addition, multivariable PI or PID controllers can be obtained with the LQR approach as demonstrated in some studies [12,15,16]. However, most plants have legacy control systems without the capacity to implement multivariable controllers, hence requiring additional hardware. Thus, multiloop PI/PID controllers, which are easier to tune and maintain that multivariable controls, remain widespread [6,9] and are still used as reference [3,10,11].

The objectives of this study are to demonstrate how to use LQR methodology to: (i) obtain multiloop proportional (P) and PI controllers and (ii) produce diagonal versions of existing multivariable PI settings obtained with different tuning methodologies.

2. Theoretical development

The infinite-horizon LQR solution for linear time-invariant (LTI) systems with full and partial state-feedback given in [17] is briefly outlined in this section. Then, LQR methodology is extended to diagonalize proportional and integral actions.

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Nomenclature

$\boldsymbol{\phi}$, $\mathbf{a}$, $\mathbf{b}_u$	auxiliary open-loop state-space matrices
$\mathbf{A}$, $\mathbf{B}_1$, $\mathbf{B}_2$, $\mathbf{C}$	open-loop state-space matrices
$\mathcal{A}$, $\mathcal{B}_1$, $\mathcal{B}_2$, $\mathcal{C}$	open-loop state-space matrices for estimating integral action
$\mathbb{A}$, $\mathbb{B}_1$, $\mathbb{B}_2$, $\mathbb{C}$, $\mathbb{D}_1$, $\mathbb{D}_2$	closed-loop state-space matrices
c	number of control variables
$\mathbf{D}$	eigenvalues of $\mathbf{K}_\mathrm{P}$ or $\mathbf{K}_\mathrm{I}$ ($\mathbb{R}^{c\times r}$, $c = r$)
$\mathbf{H}$	Hamiltonian matrix
I	quadratic performance index
J	quadratic performance index for LQR problem
$\mathbf{K}$, $\mathcal{K}$	state feedback gains in LQR problem solution
$k_.$	a given element in either $\mathbf{K}_\mathrm{P}$ or $\mathbf{K}_\mathrm{I}$
$\mathbf{k}_1$	magnitude for forcing function in regulator problem ($\mathbb{R}^r$)
$\mathbf{k}_2$	magnitude for forcing function in servo problem ($\mathbb{R}^r$)
$\mathbf{K}_\mathrm{P}$, $\mathbf{K}_\mathrm{I}$	proportional and integral gain matrices ($\mathbb{R}^{c\times r}$, $c = r$)
n	number of state variables
$\mathbf{P}$, $\mathcal{P}$	solutions to Riccati equation in LQR problem
$\mathbf{Q}$, $\boldsymbol{Q}$	output weight matrices: original ($\mathbb{R}^{r\times r}$) and integral ($\mathbb{R}^{r\times r}$), respectively
$\mathbb{Q}$	output weight matrix for closed-loop response ($\mathbb{R}^{r\times r}$)
r	number of measured output variables
$\mathbf{r}$	reference vector ($\mathbb{R}^r$)
$\mathbf{R}$, $\boldsymbol{R}$	control weight matrices: proportional ($\mathbb{R}^{r\times r}$) and integral ($\mathbb{R}^{r\times r}$), respectively
$\mathbb{R}$	control weight matrix for closed-loop response ($\mathbb{R}^{c\times c}$)
s	number of exogenous variables
$\mathbf{S}(s)$	sensitivity function matrix
$\mathbf{T}(s)$	closed-loop transfer matrix
$\mathbf{u}$, $\boldsymbol{u}$	control vectors: original ($\mathbb{R}^c$) and extended, ($\mathbb{R}^c$) respectively
$\mathbf{V}$	eigenvectors of $\mathbf{K}_\mathrm{P}$ or $\mathbf{K}_\mathrm{I}$ ($\mathbb{R}^{c\times r}$, $c = r$)
$\mathbf{x}$, $\mathcal{X}$	open-loop state vectors: original ($\mathbb{R}^n$) and extended ($\mathbb{R}^{n+r}$), respectively
$\mathbf{y}$, $\mathcal{Y}$	measured output vector: original ($\mathbb{R}^r$) and integral ($\mathbb{R}^r$), respectively
$\mathbf{w}$	exogenous signals vector ($\mathbb{R}^s$)
$\mathbf{z}$	closed-loop state vectors ($\mathbb{R}^{n+r}$)

Greek symbols

α	spectral abscissa
Δ	relative gains
ϕ	robustness index
λ	closed-loop eigenvalues
$\boldsymbol{\xi}$	integral output state ($\mathbb{R}^r$)

Subscripts

∞	final steady-state
I	for integral action
P	for proportional action
r	for regulator problem ($\mathbf{w} \neq \mathbf{0}$, $\mathbf{r} = \mathbf{0}$)
s	for servo problem ($\mathbf{r} \neq \mathbf{0}$, $\mathbf{w} = \mathbf{0}$)
ss	for steady-state
u	for control signal
y	for output signal

Superscripts

p	for an impulse forcing function ($\delta(t)$)
s	for a step forcing function ($1(t)$)

2.1. *LQR solution to LTI systems with full state and output feedback controller*

Consider the following multivariate linear time-invariant plant equipped with a PI controller ($\mathbf{r} = \mathbf{0}$)

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}_1\mathbf{w} + \mathbf{B}_2\mathbf{u}; \quad \mathbf{x}(0) = \mathbf{0} \tag{1}$$

$$\mathbf{y} = \mathbf{C}\mathbf{x} \tag{2}$$

$$\mathbf{u} = \mathbf{u}_\mathrm{P} + \mathbf{u}_\mathrm{I} \tag{3}$$

$$\mathbf{u}_\mathrm{P} = -\mathbf{K}_\mathrm{P}\mathbf{y} = -\mathbf{K}_\mathrm{P}\mathbf{C}\mathbf{x} = -\mathbf{K}\mathbf{x} \tag{4}$$

$$\mathbf{u}_\mathrm{I} = \mathbf{K}_\mathrm{I}\int_0^t (\mathbf{r} - \mathbf{y})dt = -\mathbf{K}_\mathrm{I}\int_0^t \mathbf{y}dt = \mathbf{K}_\mathrm{I}\boldsymbol{\xi} \tag{5}$$

where $\mathbf{x} = [x_1; x_2; \ldots; x_n]$, $\mathbf{w} = [w_1; w_2; \ldots; w_s]$, $\mathbf{u} = [u_1; u_2; \ldots; u_c]$, $\mathbf{y} = [y_1; y_2; \ldots; y_r]$, $\boldsymbol{\xi} = [\xi_1; \xi_2; \ldots; \xi_r]$ and $\mathbf{r} = [r_1; r_2; \ldots; r_r]$ are the state, exogenous signals, control, measured output vector, integral output and reference vectors, respectively; and $\mathbf{A} = [a_{11}\ldots a_{1n}; a_{21}\ldots a_{2n}; \ldots; a_{n1}\ldots a_{nn}]$, $\mathbf{B}_1 = [b_{111}\ldots b_{11s}; b_{121}\ldots b_{12s}; \ldots; b_{1n1}\ldots b_{1ns}]$, $\mathbf{B}_2 = [b_{211}\ldots b_{21c}; b_{221}\ldots b_{22c}; \ldots; b_{2n1}\ldots b_{2nc}]$, $\mathbf{C} = [c_{11}\ldots c_{1n}; c_{21}\ldots c_{2n}; \ldots; c_{r1}\ldots c_{rn}]$, $\mathbf{K} = [k_{11}\ldots k_{1n}; k_{21}\ldots k_{2n}; \ldots; k_{c1}\ldots k_{cn}]$, $\mathbf{K}_\mathrm{P} = [k_{\mathrm{P}11}\ldots k_{\mathrm{P}1r}; k_{\mathrm{P}21}\ldots k_{\mathrm{P}2r}; \ldots; k_{\mathrm{P}c1}\ldots k_{\mathrm{P}cr}]$ and $\mathbf{K}_\mathrm{I} = [k_{\mathrm{I}11}\ldots k_{\mathrm{I}1r}; k_{\mathrm{I}21}\ldots k_{\mathrm{I}2r}; \ldots; k_{\mathrm{I}c1}\ldots k_{\mathrm{I}cr}]$ are the state transition, disturbance, control, output, state feedback, proportional gain and integral gain matrices, respectively. LQR methodology can be used to obtain the optimal control vector $\mathbf{u}$ minimizing the following cost function (assuming $\mathbf{w} = \mathbf{0}$)

$$J(\mathbf{x}, \mathbf{u}) = \int_0^\infty \left(\mathbf{x}^\mathrm{T}\tilde{\mathbf{Q}}\mathbf{x} + \mathbf{u}^\mathrm{T}\mathbf{R}\mathbf{u}\right)dt \tag{6}$$

$$\tilde{\mathbf{Q}} = \mathbf{C}^\mathrm{T}\mathbf{Q}\mathbf{C} \tag{7}$$

where $\mathbf{Q} = [q_{11}\ldots q_{1r}; q_{21}\ldots q_{2r}; \ldots; q_{r1}\ldots q_{rr}]$ is a positive-definite (or positive-semidefinite) matrix and $\mathbf{R} = [r_{11}\ldots r_{1c}; r_{21}\ldots r_{2c}; \ldots; r_{c1}\ldots r_{cc}]$ is a positive-definite matrix. The matrices $\mathbf{Q}$ and $\mathbf{R}$ determine the relative importance of the error and the expenditure of the energy of the control signals, respectively. The LQR solution gives the optimal feedback gains

$$\mathbf{K} = \mathbf{R}^{-1}\mathbf{B}_2^\mathrm{T}\mathbf{P} \tag{8}$$

and $\mathbf{P} = [p_{11}\ldots p_{1n}; p_{21}\ldots p_{2n}; \ldots; p_{n1}\ldots p_{nn}]$ is obtained by solving the algebraic Ricatti equation (ARE)

$$\mathbf{A}^\mathrm{T}\mathbf{P} + \mathbf{P}\mathbf{A} - \mathbf{P}\mathbf{B}_2\mathbf{R}^{-1}\mathbf{B}_2^\mathrm{T}\mathbf{P} + \tilde{\mathbf{Q}} = \mathbf{0} \tag{9}$$

If $\mathbf{K}_\mathrm{I} = \mathbf{0}$ and $\mathbf{C} = \mathbf{I}$ then Eq. (3) reduces to a full state feedback controller, if $\mathbf{K}_\mathrm{I} = \mathbf{0}$ and $\mathbf{C} \neq \mathbf{I}$ the controller is known as the output feedback controller. The proportional gain matrix is estimated as

$$\mathbf{K}_\mathrm{P} = \mathbf{K}\mathbf{C}^{-1} \tag{10}$$