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Research article

The output feedback control synthesis for a class of singular fractional order systems

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ABSTRACT

This paper investigates the output feedback normalization and stabilization for singular fractional order systems with the fractional commensurate order α belonging to $(0, 2)$. Firstly, an effective criterion for the normalization of singular fractional order systems is given with output differential feedback. Afterwards, both static and dynamic output feedback stabilization of such normalized fractional order systems are derived. Besides, the robustness to the parameter uncertainty and the initial conditions are discussed in detail. All the results are given via linear matrix inequality (LMI) formulation. Finally, three numerical examples are provided to demonstrate the applicability of the proposed approaches.

1. Introduction

In recent decades, there has been a continuing growth in the number of research outcomes regarding the theoretical research and engineering applications of singular systems [1,2]. The main reason is that such systems can describe physical systems better than that of the regular systems and arise naturally in various fields including electrical networks, large-scale systems, economic systems and power systems [3]. Due to the great efforts devoted by researchers, a large volume of research work has been published on the topics of analysis and synthesis of singular systems. Besides, singular fractional order systems have been extensively studied recently due to the fact that the fractional order calculus has contributed great merits, particularly in non-short memory and non-local property for describing physical systems [4,5].

Despite these great achievements in the research about singular fractional order systems, there are still many challenging and unsolved problems in the field of stability analysis and controller synthesis [6,7]. Actually, the decision matrices have either too complex form or too many numbers in stability criterion of linear time-invariant fractional order systems [8–10]. Such limitation makes these criteria become too complicated to be used for controller design. Moreover, the fractional order Leibniz rule has infinite terms [11], which hinders the development of the Lyapunov method in fractional order case. It is worth noticing that two Lyapunov methods were established by [12,13]. The two methods have been successfully adopted in many applications [14,15], but they reduced the stability region of $0 < \alpha < 1$ to that of $\alpha = 1$ with great conservatism. It is only lately that a simple and effective LMI stability criterion has been proposed in [16] by introducing the concept of fractional order positive definite matrix. Such criterion possesses the same form as the integer order case and can be adopted to design controller conveniently.

By virtue of these stability criterion for regular fractional order systems, many theoretical results have been extended to singular cases. The admissibility conditions of singular fractional order systems with order $0 < \alpha < 2$ were first proposed by [17]. Based on his work, the related stabilization issue was further investigated by [18,19]. On the basis of a new stability criterion in [10], the admissibility condition for $0 < \alpha < 1$ case had been improved by [20]. The improved condition was adopted and applied to the design of controllers by [21]. Besides, to transform singular systems into normal ones, the normalization and stabilization for singular fractional order systems with order $0 < \alpha < 2$ was reported by [22,23]. It is a great idea to tackle the singular problem, while there is room for further investigation.

- Since it is usually not practical or even impossible to sense all the states and feed them back, it is practically and theoretically

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appealing to normalize and stabilize singular fractional order systems by output feedback technique.

- The adopted stability criterion [10] is rather complicated and it is not convenient for controller design. In general, some strong assumptions are made to get a feasible solution, which surely brings conservatism.
- The existing normalization approaches in [22,23] cannot deal with the output differential feedback problem or even are difficult to be solved via current MATLAB LMI Toolbox. Perhaps, a novel treatment is desired.
- Besides the regular static output feedback controller, the dynamic one has more design freedom on system dimension. What is more, the commensurate order of such controller might be set different from the controlled plant.
- The stability criterion in [16] only focuses the commensurate order $\alpha \in (0,1)$. It will be extremely practical to study the relation between the systems with α and 0.5α, and make it still available for the case of $(1,2)$.
- The existing works on this issue usually utilize a fact on the initial conditions directly without any proof. However, the initial value problem is a crucial one in fractional order system theory and cannot be neglected.

With the previous motivation, this paper further investigates the important issues of output feedback normalization and stabilization for singular fractional order system with $0<\alpha<2$. The remainder of this paper is organized as follows. In Section 2, some preliminaries on fractional order calculus and fractional order systems are introduced. In Section 3, the problem formulation are presented. The main results are derived strictly. Numerical simulation results that illustrate the effectiveness of the method are shown in Section 4 and the paper ends with the concluding remarks in Section 5.

Notations. In the sequel, $\mathrm{sym}(M)$ represents the expression $M^{\mathrm{T}}+M$. $*$ is used as an ellipsis for terms induced by symmetry. $\mathbb{P}_\alpha^{n\times n}$ is defined as

$$\mathbb{P}_\alpha^{n\times n}=\left\{\sin\left(\tfrac{\alpha\pi}{2}\right)X+\cos\left(\tfrac{\alpha\pi}{2}\right)Y:\, X,Y\in\mathbb{R}^{n\times n},\begin{bmatrix}X & Y\\ -Y & X\end{bmatrix}>0\right\}.$$

2. Preliminaries

Fractional order calculus can be regarded as a natural generalization of conventional integer order calculus which becomes a particularly useful tool in engineering applications. Nowadays, there are many definitions related to the different fractional order derivatives. However, up to now, there is still no global consensus for its mathematical definition and physical interpretation, especially concerning the initial value problem [24].

The widely applicable fractional order integral definition is from Riemann-Liouville integral [11]. According to this definition, the α-order integral of a function $f(t)\in\mathbb{R}$ can be expressed as

$$ {}_{t_0}\mathcal{I}_t^{\alpha}f(t)=\frac{1}{\Gamma(\alpha)}\int_{t_0}^{t}(t-\tau)^{\alpha-1}f(\tau)\mathrm{d}\tau, \tag{1}$$

with α is positive real, t_0 is the lower terminal and $\Gamma(\alpha)=\int_0^\infty x^{\alpha-1}\mathrm{e}^{-x}\mathrm{d}x$ is the well-known Gamma function.

The Caputo derivative of $f(t)$ with order α is defined by

$$\begin{aligned} {}_{t_0}\mathcal{D}_t^{\alpha}f(t)&={}_{t_0}\mathcal{I}_t^{n-\alpha}\mathcal{D}^n f(t)\\ &=\frac{1}{\Gamma(n-\alpha)}\int_{t_0}^{t}(t-\tau)^{n-\alpha-1}f^{(n)}(\tau)\mathrm{d}\tau,\end{aligned} \tag{2}$$

where $n-1<\alpha<n$ and $\mathcal{D}^n$ is the classical n-order derivative.

The Caputo definition for fractional order derivative is adopted throughout this paper, mainly for two good properties on fractional order derivative of constant and the initial value of Laplace transform. To simplify the notation, ${}_{t_0}\mathcal{D}_t^{\alpha}$ is abbreviated as $\mathcal{D}^\alpha$ when $t_0=0$. To proceed the discussion of the main results, a useful lemma will be presented for subsequent use.

Lemma 1. *[16] The n-dimensional fractional order system $\mathcal{D}^\alpha x(t)=Ax(t)$ with the order $0<\alpha<1$ is asymptotically stable if and only if there exists a matrix $P\in\mathbb{P}_\alpha^{n\times n}$, such that*

$$\mathrm{sym}(AP)<0. \tag{3}$$

Note that the eigenvalues of matrices A and A^{T} are identical. Consequently, the following conclusion can be derived easily.

Remark 1. The n-dimensional fractional order system $\mathcal{D}^\alpha x(t)=Ax(t)$ with the order $0<\alpha<1$ is asymptotically stable if and only if there exists a matrix $Q\in\mathbb{P}_\alpha^{n\times n}$, such that

$$\mathrm{sym}\left(Q^{\mathrm{T}}A\right)<0. \tag{4}$$

Furthermore, in consideration of $\cos\left(\frac{\alpha\pi}{2}\right)=0$, Lemma 1 also holds for $\alpha=1$.

3. Main Results

3.1. Normalization of singular fractional order systems

Consider the following system

$$\begin{cases} E\mathcal{D}^\alpha x(t) = Ax(t)+Bu(t),\\ y(t)=Cx(t),\end{cases} \tag{5}$$

where $u(t)\in\mathbb{R}^m$, $x(t)\in\mathbb{R}^n$ and $y(t)\in\mathbb{R}^p$ are the control input, the pseudo semi-state and the measurable output, respectively. $\alpha\in(0,1)\bigcup(1,2)$ is the system commensurate order. E is a singular square matrix. A, B and C are matrices with appropriate dimensions.

If we design the output feedback controller for system (5) in the following form

$$u(t)=-L\mathcal{D}^\alpha y(t)+v(t), \tag{6}$$

then the resulting closed-loop system can be expressed as

$$\begin{cases} (E+BLC)\mathcal{D}^\alpha x(t) = Ax(t)+Bv(t),\\ y(t)=Cx(t),\end{cases} \tag{7}$$

where $L\in\mathbb{R}^{m\times p}$ is the gain matrix to normalize the singular system (5) and $v(t)\in\mathbb{R}^m$ is the virtual control input with output feedback, which is constructed to stabilize the normalized system (7). In this study, the objective is to design suitable L and v to guarantee the asymptotic stability of resulting closed-loop control systems.

Similar to the conventional integer order case [25], the system (7) is normalized if and only if $\det(E+BLC)\neq 0$. Hence the following theorem is given to get the needed L.

Theorem 1. *The system (7) is normalized if there exist a scalar $\varepsilon\in(0,1)$ and a matrix $L\in\mathbb{R}^{m\times p}$ such that*

$$\begin{bmatrix} E^{\mathrm{T}}E+\mathrm{sym}\left(E^{\mathrm{T}}BLC+BLC\right) & I\\ I & \varepsilon I\end{bmatrix}>0. \tag{8}$$

Proof. If the inequality in (8) holds, an equivalent condition can be derived according to Schur complement operation