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Research article

Robust fast controller design via nonlinear fractional differential equations

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ABSTRACT

A new method for linear system controller design is proposed whereby the closed-loop system achieves both robustness and fast response. The robustness performance considered here means the damping ratio of closed-loop system can keep its desired value under system parameter perturbation, while the fast response, represented by rise time of system output, can be improved by tuning the controller parameter. We exploit techniques from both the nonlinear systems control and the fractional order systems control to derive a novel nonlinear fractional order controller. For theoretical analysis of the closed-loop system performance, two comparison theorems are developed for a class of fractional differential equations. Moreover, the rise time of the closed-loop system can be estimated, which facilitates our controller design to satisfy the fast response performance and maintain the robustness. Finally, numerical examples are given to illustrate the effectiveness of our methods.

1. Introduction

Overshoot and rise time of system output are two significant engineering indices to evaluate a control system performance. However, these two indices are usually conflicting with each other, which is a well-known fact. Especially, for linear plants with PID controllers, when the overshoot is compensated to a small value by tuning the PID parameters, the rise time of system response will certainly become slow, and vice versa. In fact, such phenomenon is caused by the inherence limitation of controller design, which is therefore impossible to break through by tuning the controller parameters only. Most of the existing methods trade off these two indices in order to guarantee the feasibility of a traditional controller. In view of above discussions, a nature question is that can we break through the limitation of traditional methods and develop an advanced controller that improves both the overshoot and rise time performance? This paper is denoted to this challenging problem.

In fact, many existing advanced controllers have appealing properties beyond traditional ones. To effectively promote the control system performance, nonlinear switching control method has been proposed and widely applied in control theories and engineering practices [1,2]. In particular, the well known bang-bang controller renders the closed-loop system convergence in finite time [3], that is to say, possessing fast response performance, and can be easily implemented. Meanwhile, the sliding-mode control using signed power function as feedback is recognized as one of the efficient tools to design robust controllers for complex high-order nonlinear dynamic plant operating under uncertainty conditions [4,5].

On the other hand, the fractional order systems (FOSs) have attracted lots of attention during the past few years since many engineering plants and processes that cannot be more accurately described without the introduction of fractional order calculus [6,7]. As a result of the tremendous efforts devoted by researchers, the modeling [7,8], stability analysis [9,10], controller design [11–13] and numerical approximation method [14] and so on, now involve FOSs. For robust controller design, Oustaloup has proposed the famous CRONE (Commande Robuste d'Ordre Non Entier) methodology [15], which provides a frequency-domain approach for the design of output feedback robust controllers for both integer and fractional order LTI systems. It has the reference model as Bode's ideal loop transfer function, while the purpose of CRONE controller design is to obtain an open-loop characteristic similar to that of this reference model. It was used in [16] to solve the speed control problem of multi-mass systems, while the controller permitted to ensure the robust speed control of the load in spite of plant parametric variations and speed observation errors. Morand et al. [17] had dealt with car longitudinal control performed by cruise control system, which had performed much better results in terms of robustness to mass and velocity uncertainties than the classical PI control method. The CRONE approach was also used in [18] to control the temperature of a diffusive medium. The stability robustness was guaranteed despite of variations of open-loop gain from the parametric uncertainties. However, almost all the works are limited to the linear system framework. Moreover, as limited to the inherent attributes of such linear ideal loop transfer function,

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it is difficult to improve the other index, such as the system rise time, by adjusting the controller parameters. Thus, all those researches focused only on the robustness for system uncertainties, while seldom research has dealt with the response speed of the controlled systems.

In view of the above literature review and the challenging problem we discussed, this paper proposes an innovative nonlinear control technique, named fractional order signed power feedback control, that achieves both robustness and fast response. The nonlinear control with signed power function, which is widely used in sliding-mode control, with fast response dynamic, and the framework of Bode's ideal transfer function with robust damping ratio are taken as the ideological basis of our control approach. In this paper we propose an innovative feedback control strategy that combines linear fractional order robust control as well as nonlinear feedback control in an effective way to exploit both of their advantages. We call this nonlinear control strategy the fractional order signed power feedback law.

On the other hand, although the nonlinear fractional order control is not difficult to be realized in figuration and simulation, it should be noticed that analysis of nonlinear FOSs remains extremely difficult due to the lack of any developed system theory and any efficient mathematical tool, and research on this topic is insufficient yet. For example, although the Lyapunov direct method for fractional order nonlinear dynamic systems [19] has already been proposed, fractional derivative is required for the compound Lyapunov function, which is rather complicated in most situations. Furthermore, proper Lyapunov function is still difficult to be found and the related analysis is complicated even for those integer order systems with nonlinear power feedback laws. Meanwhile, it is not an easy task to quantitatively analyze the overshoot and rise time for nonlinear systems, and seldom literatures are reported so far. Facing the great difficulties in nonlinear FOSs analysis, the comparison theorems and rise time estimation method for a class of nonlinear FOSs are proposed in this paper. Then the fractional order nonlinear system rise time can be theoretically analyzed utilizing those theorems.

The rest of this paper is arranged as follows. The next section introduces some basic definitions and the problem formulation we discussed in this paper, as well as some preliminaries that will be used in the later sections. Section 3 proposes the fractional order signed power feedback laws for both tracking and regulation problems. In order to analytically analyze the time response of such nonlinear FOS, two comparison theorems as well as rise time estimation theorems are proposed. By the table look-up, one can allocate the controller parameters to achieve desired robustness and response speed. The numerical examples are given in Section 4 to illustrate the effectiveness of the controller design method and the comparison and estimation method proposed in this paper. The main contributions and conclusions are conducted in the final section.

2. Problem formulation and preliminaries

2.1. Problem formulation

Consider a single input single output (SISO) linear system with transfer function $G(s)$, the minimum phase system $\{A, B, C\}$ with $C=1$ and B as a non-zero scalar in the range of this paper is expressed by fractional differential equations as

$$\begin{cases} D^{\alpha}x(t) = Ax(t) + B\tilde{u}(t), \\ y(t) = x(t), \end{cases} \tag{1}$$

where D^{α} is in the sense of Caputo's fractional order derivative definition as

$$D^{\alpha}f(t) = \frac{1}{\Gamma(m-\alpha)}\int_0^t \frac{f^{(m)}(\tau)}{(t-\tau)^{\alpha-m+1}}\mathrm{d}\tau, \tag{2}$$

where $m-1<\alpha<m$, $m\in\mathbb{N}$. $\tilde{u}(t)$ is the nonlinear feedback control law for $G(s)$.

Generally, system states are variables that could uniquely determine the system information at arbitrary future time along with the system input. However, as FOS is essentially an infinite dimension system [20], incompleteness problem occurs when one tries to describe the FOS by the existing definitions. That is, the finite given system states in FOS cannot reflect the whole system's information [21]. Thus, compared to the complete states in finite dimension integer order systems, those incomplete states in FOSs are called 'pseudo states'. Nevertheless, those pseudo states can also be part of real states in FOSs. In model (1), $x(t)$, $y(t)$ are the pseudo states and output of the system, respectively. In this situation $x(t)$ is also the real state that represents the system output. The real fractional order α is ranged in $(1, 2)$. In this paper the system overshoot *P.O.* is defined as the first percentage overshoot of the step response in tracking or in regulation,

$$P.O. = \frac{M_p - y_v}{y_v - y(0)} \times 100\%, \tag{3}$$

where M_p, y_v and $y(0)$ represent the output peak value, final value and the initial value respectively. The system output rise time t_r for the tracking problem is taken by $y(t)$ to change from 0 to $100p\%$ of its final value, where p is a prespecified constant arranged in $(0, 1]$.

$$t_r = \min\left\{\arg\min_{t>0}\left|y(t) - 100p\%\left[y_v - y(0)\right]\right|\right\}. \tag{4}$$

Similarly, the fall time t_f is defined for regulation problem as

$$t_f = \min\left\{\arg\min_{t>0}\left|y(t) - 100q\% y(0)\right|\right\}, \tag{5}$$

where q is constant arranged in $[0, 1)$.

Without loss of generality, those two constants are specified as $p=0.9$ and $q=0.1$ in this paper by default.

Obviously in this minimum phase system there holds $x(t) = y(t)$. Hence, state $x(t)$ represents the output $y(t)$ in the rest of this paper by default.

Our task is to design a nonlinear fractional order controller $\tilde{u}(t)$ for linear system $G(s)$ with parameter disturbance, so that the closed-loop system (1) satisfies a constant overshoot as well as a rapid rise time.

2.2. Preliminaries

As a robust control strategy, CRONE provides a fractional order approach for the robust control of uncertain plants under common unity-feedback configuration (Fig. 1) [15], where $r(t)$ is the reference input of the system, $d_u(t)$, $d_y(t)$ and $d_m(t)$ are external disturbances, and $y(t)$ is the output of the closed-loop system. For simplicity, the designing principle of second generation CRONE approach is to seek the synthesis of such an open-loop transfer function temple

$$F(s) = C(s)G(s) = \left(\frac{\omega_{cg}}{s}\right)^{\alpha} \triangleq \frac{K}{s^{\alpha}}, \tag{6}$$

where real fractional order $\alpha\in(1, 2)$ and positive system gain $K>0$. It is in fact Bode's ideal open loop transfer function, and indicates the vertical template in an open-loop Nichols locus. This vertical displacement of the template ensures the robustness of phase margin $\varphi_m = (2-\alpha)\pi/2$, which conveys the stability degree