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Research article

Passivity analysis of neural networks with two different Markovian jumping parameters and mixed time delays ☆

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ABSTRACT

This paper studies the problem of passivity analysis for neural networks with two different Markovian jumping parameters and mixed time delays utilizing some integral inequalities. The integral inequalities produce sharper bounds than what the Jensen's inequality produces, consequently, better results are obtained. The Markovian jumping parameters in connection weight matrices and discrete delay are assumed to be different in the system model. By constructing a new appropriate Lyapunov-Krasovskii functional (LKF), some sufficient conditions are established which guarantee the passivity of the proposed model. Numerical examples are given to show the less conservatism and effectiveness of the proposed method.

1. Introduction

In the past decades, neural networks have attracted considerable attention due to its wide applications in face detection, signal processing, financial industry, motion control and so on [1–4]. In real applications of neurons, since the finite conduction speed of action potential or finite signal propagation time in the process of information transformation, time delay referred as a typical characteristic of the signal transmission between neurons. That is, the law of system development is not only depends on the current states, but also depends on the previous states. In fact, many oscillation behaviors, bifurcation and even instability cases are resulted from time delays. Hence, there are lots of graceful results about the stability of neural networks with different types of delays (discrete delay, distributed delay and leakage delay) [5–9]. Especially, leakage delay [8,9] should be added to the considered neural networks which exists in the negative feedback term of the system and destabilizes the dynamical behaviors of system.

As we all know, neural networks with Markovian jump parameters [10–12] can be used to model some practical systems where they may experience abrupt changes in their structures and parameters, possibly caused by repairs of components, changing subsystem interconnections and sudden environmental disturbances. In general, this class of system has finite modes and the switching between these modes is determined by a Markov chain [13–15]. The dynamic behaviors of Markovian jumping neural networks with discrete delay [16–19], distributed delay [18–20] or leakage delay [21–23] have been investigated. The state estimation problem for a new class of discrete-time neural networks with Markovian jumping parameters as well as mode-dependent mixed time-delays is studied in [24] by constructing a novel LKF. The authors in [25] consider the robust stability problem for a class of discrete-time uncertain Markovian jumping neural networks with defective statistics of modes transitions. In [26], Chandrasekara and Rakkiyappan study the impulsive synchronization of Markovian jumping randomly coupled neural networks with partly unknown transition probabilities via multiple integral approach. However, in existing works, either the time delay is irrelated to Markovian jumping parameters [16–23,26] or the Markovian jumping parameters in time delay are the same as ones in connection weight matrices [17,24]. Actually, due to the dynamic systems subject to abrupt variation frequently in their structures, the time delay may also have finite modes and the switching between different

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time delay modes can also be governed by a Markovian chain, which is different from the one in connection weight matrices. Thus, it is essential to study neural networks with two different Markovian jumping parameters and time delays.

Passivity performance [27] is another critical problem of neural networks, which is intimately related to the circuit analysis method [28]. Since the passive property can keep a system internal stability, it plays an important role in the analysis of the stability of dynamical systems for nonlinear control and other research fields. In the field of passivity analysis, how to obtain a tighter bounds of integral terms is a crucial step in reducing the conservatism. There have been many approaches in the existing literature such as delay decomposition approach [29], convex optimization method [30], free-weighting matrix approach [32,33] and so on. Recently, lots of papers have adopted the Jensen's inequality [31,34–37] which requires fewer decision variables than other existing methods having a better performance behavior. However, as pointed in [38], the Jensen's inequality usually produces an undesirable conservatism by using the Gruss inequality. Furthermore, a alternative inequality so-called Wirtinger-based integral inequality obtaining a tighter upper bound of integral terms has been developed in [39] based on the extended Wirtinger inequality. Moreover, a novel integral inequality, namely Bessel-Legendre (B-L) inequality is proposed in [40] which encompasses the Jensen's inequality and the Wirtinger-based integral inequality. Nevertheless, the inequalities in [39–41] are unable to process double integral terms of quadratic functions. Fortunately, The new inequality is discussed in [42] reducing the gap of the inequalities in [39–41]. But so far, the improved inequalities in [42] have not been applied to the delay neural networks with Markovian jumping parameters.

Besides improving inequality-based techniques in order to estimate more accurately the derivatives of the constructed Lyapunov-Krasovskii functionals (LKFs), constructing an appropriate LKF is also a key part in reducing the conservatism. A typical Lyapunov functional is constructed in [43] where only two matrices P_i and Q_i are mode-dependent. Due to mode-dependent matrices (distinct matrices are chosen for different modes), are more flexible than the common ones, the problem of state estimation of recurrent neural networks with Markovian jumping parameters and mixed delays is further investigated in [44] by constructing a novel Lyapunov functional, which makes as many as possible of the Lyapunov matrices are chosen to be mode-dependent. However, the information of states is not used to their full potential. The triple integrals are used to construct the LKF in [44], but the domain of integration only include the lower region of regular triangular prism. If the upper region of regular triangular prism can be also included as the domain of integration, a less conservative condition may be obtained. This is because more information of states is considered.

Motivated by these considerations, the problem of passivity analysis for neural networks with two different Markovian jumping parameters and multiple time delays is investigated in this paper. A new weak infinitesimal operator is proposed to act on LKF with two different Markovian jumping parameters. By constructing a new appropriate LKF and combining with the new integral inequalities, which obtain a sharper bounds of integral terms than what the Jensen's inequality produces. some less conservatism mode-dependent criteria are established to guarantee the passivity of the considered model. Numerical examples are given to show the effectiveness of the proposed method.

Notations: Throughout this paper, the superscripts '-1' and 'T' stand for the inverse and transpose of a matrix, respectively; $P > 0$, ($P \geq 0$, $P < 0$, $P \leq 0$) means that the matrix P is symmetric positive definite(positive-semi definite, negative definite and negative-semi definite); R^n denotes n-dimensional Euclidean space; $R^{m\times n}$ is the set of $m \times n$ real matrices; The identity matrix of order n is denoted as I_n; $*$ denotes the symmetric block in symmetric matrix; $\lambda_{\max}(Q)$ and $\lambda_{\min}(Q)$ denote, respectively, the maximal and minimal eigenvalue of matrix Q; $\mathbf{E}$ stands for the corresponding expectation operator.

2. Problem statement and preliminaries

Consider neural networks with multiple delays and two Markovian jumping parameters

$$\begin{cases} \dot{x}(t) &= -C(r_t)x(t-\sigma) + W_0(r_t)g(x(t)) + W_1(r_t)g(x(t-h(t,\delta_t))) + W_2(r_t)\int_{t-d}^{t} g(x(s))ds + u(t), \\ y(t) &= D(r_t)x(t) + E(r_t)\tilde{g}(t, x(t)), \end{cases} \tag{1}$$

where $x(t) = [x_1(t), x_2(t), \ldots, x_n(t)]^T \in R^n$ is the neural state vector; $g(x(t)) = [g_1(x_1(t)), g_2(x_2(t)), \ldots, g_n(x_n(t))]^T$ represents the neuron activation function; $\tilde{g}(t, x(t))$ is a nonlinear disturbance function; $y(t) \in R^m$ is the network measurement; $u(t)$ is a external input vector; scalars $\sigma \geq 0$ and $d \geq 0$ designate leakage and distributed delays, respectively; $C(r_t) = diag\{c_1(r_t), c_2(r_t), \ldots, c_n(r_t)\} > 0$ representing the firing rate, $W_0(r_t)$, $W_1(r_t)$, $W_2(r_t)$, $D(r_t) \in R^{m\times n}$ and $E(r_t) \in R^{m\times m}$ are the connection weight matrices functions of the random jumping process r_t, where r_t is a finite-state Markovian jump process representing the system mode, i.e., r_t takes discrete values in a given finite set $\varsigma_1 = \{1, 2, \ldots, N_1\}$. $h(t, \delta_t)$ is a discrete mode-dependent time-varying delays, where δ_t is another different Markovian jump process, which takes discrete values in $\varsigma_2 = \{1, 2, \ldots, N_2\}$. The transition probability matrices $\Pi = (\pi_{ij})_{N_1\times N_1}$ and $P = (p_{kl})_{N_2\times N_2}$ of system (1) are given by

$$pr(r_{t+\vartriangle} = j | r_t = i) = \begin{cases} \pi_{ij}\vartriangle + o(\vartriangle), & j \neq i \\ 1 + \pi_{ii}\vartriangle + o(\vartriangle), & j = i \end{cases},$$

$$pr(\delta_{t+\vartriangle} = l | \delta_t = k) = \begin{cases} p_{kl}\vartriangle + o(\vartriangle), & l \neq k \\ 1 + p_{kk}\vartriangle + o(\vartriangle), & l = k \end{cases},$$

where $\vartriangle > 0$, $\lim_{\vartriangle\to 0}\frac{o(\vartriangle)}{\vartriangle} = 0$, $\pi_{ij} \geq 0$, $\forall\, j \neq i$, is the transition rate from mode i at time t to mode j at time $t + \vartriangle$, and $\pi_{ii} = -\sum_{j=1, j\neq i}^{j=N_1}\pi_{ij}$. $p_{kl} \geq 0$, $\forall\, k \neq l$, is the transition rate from mode k at time t to mode l at time $t + \vartriangle$, and $p_{kk} = -\sum_{l=1, l\neq k}^{l=N_2} p_{kl}$.

Remark 1. In existing works, either the time delay is irrelated to Markovian jumping parameters [17–25,27] or the Markovian jumping parameters in time delay are the same as ones in connection weight matrices [18]. Such as, a typical Markovian jumping neural network