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Robust absolute stability analysis for interval nonlinear active disturbance rejection based control system

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ABSTRACT

Active disturbance rejection control (ADRC) is a relatively new, quite different but very practical control technology, which shows much promise in replacement of proportion-integration-differentiation (PID) with unmistakable advantage in performance and practicality. This paper mainly concerns with the robust absolute stability of the ADRC based control system with parameter perturbations of the plant, i.e., ADRC based interval control system. Firstly, the system is transformed into a perturbed indirect Lurie system. Then, the Popov criterion and its robust version are presented and some new methods are developed to analyze the (robust) absolute stability for the interval control system. Furthermore, an example is presented to illustrate (robust) absolute stability analysis via the above methods, which verifies the convenience and practicability of these methods and shows the strong stability robustness of ADRC in the presence of parametric uncertainties.

1. Introduction

Active disturbance rejection control (ADRC) is a new practical control technique, which was systematically proposed by Professor Han, a famous Chinese scholar, and published in Chinese in 1998 [1]. Han's works [2–5,1,6] were previously published in Chinese and not widely accessible by western countries. Though sporadical works about its ideas [7–9] and applications appeared in the English literature before, but ADRC was not fully published in English [10] until 2009. ADRC can be summarized as follows: it is almost error driven and less dependent on the model, which inherits from the most widely used proportional-integral-derivative (PID) today; it makes use of an extended state observer (ESO) to estimate and compensate the internal and external disturbances and uncertainties; it embraces the power of nonlinear feedback and puts it to full use; it is a useful digital control technology developed out of an experimental platform rooted in computer simulations. ADRC has been a work in progress for almost two decades, more and more works on its theoretical analysis and applications in the English literature have appeared in recent years; see, for example, [12,11,13–19].

ADRC was firstly developed as nonlinear structure by Professor Han and latter linearized and parameterized by Professor Gao [20], a cooperator of Han. The nonlinear ADRC prefers to use nonlinear functions in the design of the observer and the control law, which is potentially much more effective in tolerance on uncertainties and disturbances and improvement of system dynamics. However, it may make the system produce some complex but colorful nonlinear behaviors, such as multiple equilibrium points, limit cycles, bifurcations and chaos. The linear ADRC is superior to the nonlinear ADRC in parameters tuning and theoretical analysis on performance and stability. Since the linear ADRC was proposed, it greatly promoted the theoretical analysis and application of ADRC. As a matter of fact, the linear ADRC is competent in most occasions. However, if there is a need to pursue more efficiency and an easy way to tune parameters of the controller and analyze the performance and stability of the nonlinear ADRC, we tend to use the nonlinear ADRC. In other words, we adopt the nonlinear ADRC or linear ADRC according to our needs.

This paper focuses on the stability analysis of the nonlinear ADRC based control system. In fact, there has already been some research results about this topic. In [21,22], time domain convergences of the nonlinear ADRC based control system were proved some sufficient conditions were presented. However, it is difficult to deal with a general nonlinear ADRC based control system, due to too many constraints and complex derivation process. In [23,24], stability analysis of the nonlinear ADRC based control system was performed via the describing function method. However, the limitations of the describing function method are apparent, and it is complicated and not universal to transform the

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nonlinear ADRC system into a system suitable for the describing function method by graphics transformation. Meanwhile, in [25–28], the time domain stability analysis of the linear ADRC based control system was proved, which revealed the relationship between the bandwidth and performance, but the method may not be direct to judge the stability of a practical system. In [29–32], the frequency domain stability analysis of the linear ADRC based closed-loop system was performed with consideration of the parameter perturbations of the linear plants simultaneously.

Due to the widespread presence of parameter perturbations of the plants (i.e., interval plants), it is necessary to perform quantitative robust stability analysis for the nonlinear ADRC based control system, which still has not been effectively solved yet. As for the interval plants, there are many results about the robust absolute stability of the Lurie system, which consists of a linear dynamics in the feedforward with nonlinear feedback constrained by a certain sector bound. There are mainly two kinds of methods to analyze the robust absolute stability of the interval Lurie system, i.e., frequency domain method [33] and time domain method [34]. This paper mainly concerns with the (robust) absolute stability of the interval ADRC based control system via the (robust) Popov criterion and some newly developed methods. The main contributions are as follows:

- i The nonlinear ADRC based control system is transformed into an indirect Lurie system, to which the absolutely stability theory can be applied.
- ii Some new methods for the stability and robust stability of the indirect Lurie system are proposed. Compared with the Popov criterion, it may be more convenient and less conservative.
- iii The new methods along with the (robust) Popov criterion are applied to the stability and robust stability of the nonlinear ADRC based control system, which also reveals its strong robust stability.

The rest of this paper is organized as follows. Section 2 introduces the framework and algorithm of ADRC, and transforms the ADRC based control system into an indirect Lurie system. In Section 3, relative notations, the Popov criterion and the robust Popov criterion are presented. Then some new theorems are developed to analyze the (robust) absolute stability for interval Lurie systems in Section 4. In Section 5, an example is presented to illustrate the application of the above methods on stability analysis. Finally, some concluding remarks are drawn in Section 6.

2. Problem formulation

2.1. Nonlinear ADRC algorithm and framework

ADRC can integrally and effectively deal with various nonlinearities, uncertainties and disturbances via estimation and compensation by ESO, which will be illustrated latter. However, it is very difficult to perform theoretical analysis, due to lack of specific mathematical description of uncertain dynamics. Although some results have been presented in the introduction, they may not be direct to practical application. This paper mainly studies the robust stability with the parameter perturbations of the plants, and considers the following typical single-input single-output (SISO) system described as

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 \\ \vdots \\ \dot{x}_n = a_n x_1 + a_{n-1}x_2 + \cdots a_1 x_n + b_0 u \\ y = x_1 \end{cases} \tag{1}$$

where y is the regulated output, u is the input force, b_0 is the gain coefficient, $x_i(i=1,2,\ldots n)$ are the system states, $a_i(i=1,2,\ldots n)$ are the gain coefficients of the system states, and $a_n x_1 + a_{n-1}x_2 + \cdots a_1 x_n$ is treated as total disturbances.

Assumption A1. The parameter perturbations of the plant (1) meet the following conditions:

$$a_I = \{a_i\colon \underline{a}_i \le a_i \le \bar{a}_i,\ i = 1, 2\ldots, n;\ a_i \in R,\ \underline{a}_i \in R,\ \bar{a}_i \in R\} \tag{2}$$

For the above plant (1), an ADRC controller can be designed. ADRC generally consists of a tracking differentiator (TD), an ESO and the state error feedback control law (SEF), which will be simply introduced as follows.

TD is used to generate the arranged transition process and extract the input signal's differential of each order. Referring to [1], a nonlinear TD is designed in the form of

$$\begin{cases} \dot{v}_1 = v_2 \\ \dot{v}_2 = v_3 \\ \vdots \\ \dot{v}_{n-1} = v_n \\ \dot{v}_n = \lambda^n \psi\left(v_1 - r, \dfrac{v_2}{\lambda}, \ldots, \dfrac{v_n}{\lambda^{n-1}}\right) \end{cases} \tag{3}$$

where r is the input force, v_i $(i = 1, 2, \ldots, n)$ is the output, λ is the adjustable speed factor, $\psi\left(v_1 - r, \frac{v_2}{\lambda}, \ldots, \frac{v_n}{\lambda^{n-1}}\right)$ is a solution that guarantees the fast convergence from v_1 to r.

ESO is used to estimate and compensate the total disturbances, which is the core and essence of the ADRC. Referring to [1], a nonlinear ESO is designed in the form of

$$\begin{cases} e = z_1 - y \\ \dot{z}_1 = z_2 - \beta_1 \cdot \varphi(e) \\ \dot{z}_2 = z_3 - \beta_2 \cdot \varphi(e) \\ \vdots \\ \dot{z}_n = z_{n+1} - \beta_n \cdot \varphi(e) + b_0 \cdot u \\ \dot{z}_{n+1} = -\beta_{n+1} \cdot \varphi(e) \end{cases} \tag{4}$$

where the inputs of the ESO are the output (y) and the control law (u) of the plant; the outputs of the ESO are $z_i(i = 1, 2, \ldots, n+1)$, and z_{n+1} provides an estimation of the total disturbances $a_n x_1 + a_{n-1}x_2 + \cdots a_1 x_n$; $\beta_i(i = 1, 2, \ldots, n+1)$ are the observer gains, e is the observer error. $\varphi(e)$ are nonlinear functions, particularly, defined as [35]

$$\varphi(e) = \mathrm{fal}(e, \alpha, \delta) = \begin{cases} (\alpha-1)\delta^{\alpha-3}e^3 - (\alpha-1)\delta^{\alpha-2}e^2\mathrm{sgn}(e) + \delta^{\alpha-1}e, & |e| \le \delta \\ |e|^\alpha \mathrm{sgn}(e), & |e| > \delta \end{cases} \tag{5}$$

where α and δ are two important parameters to be predetermined. The original version of the $\mathrm{fal}(e, \alpha, \delta)$ function was firstly proposed by Han [6], and this continuous and smooth $\mathrm{fal}(e, \alpha, \delta)$ function was an improved inversion to avoid oscillation. It plays an important role in the newly proposed ADRC framework, due to its characteristics of 'small error, big gain; big error, small gain' with $\alpha < 1$. $\mathrm{fal}(e, \alpha, \delta)$ is denoted as $\mathrm{fal}(e)$ throughout this paper.

SEF is used to restrain the residual error and achieve the desired control goal. The control law is designed as