ISA Transactions ∎ (∎∎∎∎) ∎∎∎–∎∎∎

Contents lists available at ScienceDirect

ISA Transactions

journal homepage: www.elsevier.com/locate/isatrans

Research article

Multi-formation control of nonlinear leader-following multi-agent systems ☆

Tao Han [a,b], Zhi-Hong Guan [a,*], Ming Chi [a,*], Bin Hu [a], Tao Li [c], Xian-He Zhang [b]

[a] College of Automation, Huazhong University of Science and Technology, Wuhan 430074, PR China
[b] College of Mechatronics and Control Engineering, Hubei Normal University, Huangshi 435002, PR China
[c] School of Electronics and Information, Yangtze University, Jingzhou 434023, PR China

ARTICLE INFO

Article history:
Received 11 October 2016
Received in revised form
26 March 2017
Accepted 1 May 2017

Keywords:
Multi-formation control
Nonlinear dynamics
Leader-following multi-agent systems
Switching topology

ABSTRACT

This paper deals with the multi-formation control problem for nonlinear leader-following multi-agent systems. Both the fixed topology case and the switching topology case are considered. The neighbor-based multi-formation control protocols are proposed under the assumption that for one subgroup, the total information received from other subgroups is zero. Then, based on the Lyapunov stability theory combined with the algebraic graph theory, sufficient conditions are established to ensure that the leader-following multi-agent systems with nonlinear dynamics can reach and maintain the desired multi-formation control. Finally, simulation examples are provided to illustrate the effectiveness of the theoretical results.

1. Introduction

The last decade has witnessed the rapid developments of the distributed cooperative control problems in multi-agent systems. The main reason is due to its important practical applications, such as cluster of satellites, flocking control of mobile robots, formation control of unmanned air vehicles, attitude alignment of spacecrafts and so on. As one of the most significant issues in cooperative control of multi-agent systems, formation control has been extensively investigated via various control approaches, namely, behavior based approach [1,2], leader-follower based approach [3,4], virtual structure based approach [5,6], and graph based approach [7,8].

Formation control has been a significant application of the control systems, such as stochastic system, networked control system [9] and so on. The aim of the formation control problem is to design appropriate protocols such that a group of agents can form a desired geometric structure and maintain it over time. Recently, based on the communication topology and sensing capability of mobile agents, many consensus-based control protocols have been proposed to systematically study the formation control of multi-agent systems. A distributed coordination scheme with local information exchange for multiple vehicle systems was described in [10], and consensus protocols were applied to the formation control of multiple mobile robots. Xie et al. [11] proposed a decentralized feedback controller for a group of mobile robots to asymptotically converge to a given moving formation. Oh et al. [12] presented a position estimation based formation control strategy for single-integrator multi-agent systems by using relative position measurements. Zhang et al. [13] solved the ultra-fast formation control problem of high-order discrete-time multi-agent systems and a novel ultra-fast protocol with self-feedback term was proposed by using the local neighbor-error knowledge. Dong et al. [14] discussed the time-varying formation analysis and design problems for general linear multi-agent systems under switching directed interaction topologies. Xia et al. [15] investigated the formation control and collision avoidance problems for multi-agent systems and a consensus protocol based on position estimation was presented to ensure the agents converge to the formation in a cooperative manner.

It is worthwhile to mention that all results above only considered normal formation, i.e., all the agents converge to the same geometric formation, while neglected the mobile agents can also achieve different kinds of formation to accomplish different tasks in an efficient way. Actually, the phenomena that different agents complete various formation shapes to implement different tasks are ubiquitous in nature and human society, such as the dispersion formation of mobile robots for multiple tasks, the predation formation of predators for multi-prey enclosing, and the division formation of social communities for multiple interests and conflicts. Therefore, it is necessary and important to study the multi-formation control problem, which means the multi-agent systems can be divided into multiple subgroups, different subgroup can

☆This work was supported in part by the National Natural Science Foundation of China under Grants 61473128, 61672245, 61370093, 61572208, 61672112, and 61672244.

* Corresponding authors.
E-mail addresses: zhguan@mail.hust.edu.cn (Z.-H. Guan), chiming@hust.edu.cn (M. Chi).

http://dx.doi.org/10.1016/j.isatra.2017.05.003

reach different desired sub-formation, and agents in the same subgroup converge to a uniform sub-formation.

Nevertheless, as far as we know, there are few results on the multi-formation control problem to date and many recent research results mainly focus on group consensus [16–18], cluster consensus [19–21], multi-consensus problem [22–24] and multi-tracking problem [25,26], where a network of agents evolve into several subgroups and agents in the same subgroup can reach the same consistent state asymptotically. Group consensus in multi-agent systems with switching topologies and communication delays was studied in [16]. By a distributed feedback controller, the cluster consensus control for generic linear multi-agent systems with directed interaction topology was investigated in [20]. The impulsive multi-consensus problems for second-order multi-agent networks using sampled position data were solved in [23]. In the light of impulsive control method, necessary and sufficient conditions on feedback gains and sampling period were provided in [25] to achieve the multi-tracking of second-order multi-agent systems.

Moreover, in practice, owing to the influence of other agents dynamics and own intrinsic dynamics, the mathematical model of agents should be introduced a nonlinear term to describe the intrinsic dynamics of each agent. Hence, it is important and meaningful to further study the multi-formation control for multi-agent systems with nonlinear dynamics. Meanwhile, in some practical circumstances, due to the disturbances of complex environment and the communication range limitations of sensors, multi-formation control problem with switching topology is considered, which is more challenging and complicated than the fixed case.

Enlightened by all the discussions above and our preliminary works on distributed cooperative control [22–25,27–29] and networked control systems [33,34], the paper aims to address the multi-formation control problem for leader-following multi-agent systems with nonlinear dynamics, which means that agents in the same subgroup can reach a desired leader-following sub-formation and there exist multiple leader-following sub-formations in the nonlinear multi-agent systems. In this paper, the multi-formation control problems are solved with the fixed and switching communication topologies, respectively. Based on the Lyapunov stability theory combined with the algebraic graph theory, the neighbor-based multi-formation control protocols are proposed under fixed and switching topologies. Then, by assuming that the total information of one subgroup receiving from other subgroups is zero, sufficient conditions are obtained such that the nonlinear leader-following multi-agent systems can achieve and maintain the multi-formation control. In summary, the main contributions of this paper are listed as follows. (i) Compared with the existing papers about the normal formation control, we consider multi-formation control problem, which is more practical and challenging, and also provides more flexibility, stability and robustness for the agents to accommodate the changes of environments, situations and distributed cooperative tasks. (ii) In an analysis of the multi-formation control problem, the leader-following approach is employed, which is simple, reliable and without the requirement of global information. Then, each subgroup is assigned with a leader, and all the followers in the same subgroup can track the corresponding leader with desired sub-formation. Specially, if the leader is dynamic, how to design a multi-formation control protocol becomes more interesting and challenging. (iii) The multi-agent systems with nonlinear dynamics are considered in the paper, where a nonlinear term is introduced to describe the intrinsic dynamics of each agent. In addition, switching communication topology is taken into consideration. Evidently, the multi-formation control problem with nonlinear dynamics and switching topology is complicated and significant in theory and application.

The rest of this article is organized as follows. In Section 2, some basic notations, related definitions, lemmas and the problem formulation are briefly presented. The multi-formation control problems under fixed topology and switching topology are discussed in Section 3, respectively. In Section 4, two numerical examples are given to illustrate the effectiveness of the theoretical results. Finally, concluding remarks are given in Section 5.

2. Preliminaries and problem formulation

2.1. Notations

The following mathematical notations used throughout this paper are fairly standard. Let $\mathbb{R}^n$ be the n-dimensional Euclidean space, $\mathbb{R}^{m\times n}$ be the space of $m \times n$ real matrices. I_n represents the $n \times n$ real identity matrix. Denote $\mathbf{1}_n$ and $\mathbf{0}_n$ as the $n \times 1$ column vector of all ones and all zeros, respectively. The symbol $\otimes$ is the Kronecker product. A^T stands for the transpose of matrix A. A is called a nonnegative matrix, denoted by $A \geq 0$, if all its entries are nonnegative. Let $diag(A) = diag(a_1, a_2, \dots, a_n)$ represent a diagonal matrix with $A = \left[a_1, a_2, \dots, a_n\right]^T \in \mathbb{R}^n$.

2.2. Graph theory

We consider leader-following multi-agent systems consisting of N follower agents and M leaders. The interaction topology of N followers is denoted by a weighted undirected graph $\mathcal{G} = \{\mathcal{V}, \mathcal{E}, \mathcal{A}\}$, where $\mathcal{V} = \{1, \dots, N\}$, $\mathcal{E} \subset \mathcal{V} \times \mathcal{V}$, and $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{n\times n}$ are a vertex set, a link set and a nonnegative weighted adjacency matrix, respectively. For an undirected graph, $(i, j) \in \mathcal{E}$ if and only if $a_{ij} = a_{ji} > 0$, which implies agent i and agent j can interact with each other. It is assumed that there are no self-loops, i.e., $a_{ii} = 0$ for all $i \in \mathcal{V}$. The degree matrix $\mathcal{D} = diag\left(d_1, d_2, \dots, d_N\right)$ is a diagonal matrix, where diagonal elements $d_i = \sum_{j=1}^{N} a_{ij}$ for $i \in \mathcal{V}$. Then, the Laplacian matrix of the weighted undirected graph $\mathcal{G}$ is defined as $\mathcal{L} = \left[l_{ij}\right]_{N\times N} = \mathcal{D} - \mathcal{A}$, where $l_{ii} = \sum_{j=1}^{N} a_{ij}$ and $l_{ij} = -a_{ij}$, $i \neq j$. If there is a path between any two vertices of a graph $\mathcal{G}$, then $\mathcal{G}$ is connected, otherwise disconnected.

In what follows, to study a leader-following problem, a graph $\bar{\mathcal{G}}$ is employed to denote the communication topology between the N followers (labeled $1, \dots, N$) and the M leaders (labeled 0). Meanwhile, the connection weight between agent i and the leader is denoted by b_i, where if the ith follower is connected to the leader, then $b_i > 0$, otherwise $b_i = 0$. The corresponding Laplacian matrix of the graph $\bar{\mathcal{G}}$ is $\bar{\mathcal{L}} = \mathcal{L} + B$ with $B = diag\left(b_1, b_2, \dots, b_N\right)$.

2.3. Problem formulation

Consider a leader-following multi-agent system with $N+M$ agents. The dynamics of the ith follower is described by

$$\begin{cases} x_i(t) = v_i(t) \\ v_i(t) = u_i(t) + f(v_i(t), t),\ i \in \mathcal{V}, \end{cases} \tag{1}$$

where $x_i(t) \in \mathbb{R}^n$, $v_i(t) \in \mathbb{R}^n$ and $u_i(t) \in \mathbb{R}^n$ denote the position, velocity and control input of agent i, respectively. $f(v_i(t), t)$ is an nonlinear continuously differentiable vector-value function, which describes the intrinsic dynamics of agent i. The dynamics of the leader can be described in the following form

$$\begin{cases} \dot{x}_j^0(t) = v_j^0(t), \\ \dot{v}_j^0(t) = f\left(v_j^0(t), t\right), j \in \left\{1, 2, \dots M\right\}, \end{cases} \tag{2}$$

where $x_j^0(t) \in \mathbb{R}^n$ and $v_j^0(t) \in \mathbb{R}^n$ are the position and velocity of the leader, respectively, and $f(v_j^0(t), t)$ is the control input. In this

Please cite this article as: Han T, et al. Multi-formation control of nonlinear leader-following multi-agent systems. ISA Transactions (2017), http://dx.doi.org/10.1016/j.isatra.2017.05.003