ARTICLE IN PRESS

ISA Transactions ■ (■■■■) ■■■–■■■

Contents lists available at ScienceDirect

ISA Transactions

journal homepage: www.elsevier.com/locate/isatrans

Stochastic stability and stabilization of Markov jump linear systems with instantly time-varying transition rates: A unified framework

Mona Faraji-Niri [1], Mohammad-Reza Jahed-Motlagh *

Department of Electrical Engineering, Control Faculty, Iran University of Science and Technology, Tehran, Iran

ARTICLE INFO

Article history:
Received 11 January 2016
Received in revised form
13 June 2016
Accepted 23 June 2016

Keywords:
Non-Homogeneous Markov Jump Linear System
Time-Varying Transition Rates
Stochastic Stability
Stabilization
Lyapunov Function
Data Clustering

ABSTRACT

This paper investigates the stochastic stability and stabilization problems of non-homogeneous Markov jump linear systems (NHMJLSs) characterized by instantly unconditionally time-varying transition rates (TRs). The novelty of the study lies in proposing a systematic method for achieving finite dimensional conditions with an acceptable degree of conservativeness for the stability and the stabilization problems of the system. In this framework, by first processing the time-varying TRs, a finite number of uncertain but time-constant TR matrices are obtained. Then, a high-level switching signal is constructed for the system, which models the contribution of each possible time-constant TR matrix. Based on the results, the NHMJLS is reformed into an uncertain switching structure referred to as the associated switched Markov jump linear system (AS-MJLS). Finally, by taking advantage of the new representation, sufficient conditions are obtained to ensure the stability and stabilizability of the system, also the controller gains are designed. The proposed framework provides a realistic representation as well as practically solvable analysis and synthesis conditions for the NHMJLS. It also leads to less conservative results compared with the existing well-known techniques. Comparative simulation studies for a single-machine infinite-bus (SMIB) power system subject to stochastically varying load demonstrate the efficiency and superiority of the method.

1. Introduction

Over the past three decades, Markov jump linear systems (MJLSs) have received a great deal of attention [1–4]. They have developed rapidly due to their ability to characterize random and abrupt changes in system dynamics induced by random faults, failures, repairs and unexpected configuration conversions [5]. MJLSs consist of a set of operation modes described by differential equations and a finite state Markov process [6] that governs the jump between these modes. The most dominant factor of an MJLS is the transition rates of the jump process, which determines the behavior of the system. Remarkably, the transition rate matrix is a completely different concept from the state transition matrix. While the TR matrix is an array of numbers describing the rate a continuous time Markov chain moves between states, the state-transition matrix is a matrix used to obtain the general solution of linear dynamical systems [7,8]. Generally, if the TRs remain constant over time, the system is called a time-homogeneous Markov jump linear system (HMJLS); otherwise, it is a non-homogeneous or time-inhomogeneous Markovian structure [6]. In the past few decades, based on the assumption that the TRs are time-invariant, MJLSs have been studied quite extensively, and valuable results have been obtained for the stability, stabilization and controller design [1–6,9–11] of these systems.

Recently, a new research tendency is to investigate MJLS with time-varying transition rates. The motivation for the study is the potential of NHMJLSs to reduce the intrinsic conservativeness of the homogeneous MJLSs in representing the reality of practical systems [12]. For example, consider economic systems where the state of the economy is roughly categorized as one of three possible operation modes (normal, boom and slump), with the evolution between them modeled by a Markov process [10,13]. In reality, the movement probabilities of the jump process are influenced by many external factors, including economic demands, cost factors, government policies and economic fundamentals [13]. In fact, the resulting transition probability matrix of the system will be distinct over different periods of time. Therefore, modeling this system as a non-homogeneous MJLS is a more realistic scenario compared to representing it as a homogeneous Markovian structure. There exist various references regarding non-homogeneous Markovian structures that investigate the stability and stabilization [14–16], control [17–19], estimation [12,20], filtering [21,22] and fault detection [23] problems of such systems.

* Corresponding author. Tel.: +98 21 77440540.
E-mail addresses: M_farajiniri@iust.ac.ir (M. Faraji-Niri), Jahedmr@iust.ac.ir (M.-R. Jahed-Motlagh).
[1] Tel.: +98 21 22509298.

http://dx.doi.org/10.1016/j.isatra.2016.06.011
0019-0578/© 2016 ISA. Published by Elsevier Ltd. All rights reserved.

However, all of the results suppose piecewise-constant TRs or TRs enclosed by a polytope [12,14–23], which mean that the TRs are varying but invariant in an interval. This critical assumption is very conservative and limits the non-homogeneous Markovian jump structures to a class of MJLSs with locally time-constant TRs. Up to now, the issues of Markovian systems with instantly unconditionally time-varying TRs have not been fully investigated. In fact, although, there exist some results developed in the purely mathematical field of the MJLSs with continuously time-varying TRs [14,24,25], but they generally involve infinite dimensional mathematical criteria (one criterion at each time point) with an evident lack of practical techniques to implement.

Taking into account the above discussions, the main objective of the present research is to investigate the stability and control concepts of the MJLSs subject to instantly time-varying TRs to develop computationally effective results with an acceptable degree of conservativeness.

The contribution of this study is twofold. First, the obstacle of the instantly time-varying transition rates confronted in system analysis and synthesis is addressed using data clustering [26,27]. The clustering procedure allows one piece of TR data to belong to two or more clusters and helps to approximate the time-varying TR signal with a finite valued signal. Based on the partitioned TR data, a new switching signal and a set of possible time-constant TR matrices are obtained for the original system. Then, the NHMJLS is reformed into a switching structure called the associated switched Markov jump linear system. The new representation is subject to two levels of switching; high-level and low-level. The high-level switching signal is constructed by the combination of the partitioned TR signals, and it specifies the active time-constant TR matrix at each time point. The low-level signal switches between system dynamic modes and is an outcome of the Markov process related to each time-constant TR matrix. To take on the responsibility of the uncertainty raised by representing the NHMJLS by the AS-MJLS, the difference between the actual and the partitioned TRs, which is called here the partitioning error (PError), is modeled as additive time-dependent but norm-bounded uncertainty for each possible TR matrix. The new representation is advantageous over the previous ones [12,14–23] because it is obtainable through a systematic approach with the ability to predefine the precision. The precision of the proposed representation is easily verifiable by comparing the mean residence time in each mode of the represented and original systems.

The second aspect of the contribution is establishing computationally effective sufficient conditions for the stability and stabilizability analysis of the system using the developed AS-MJLS. The analysis and synthesis methodology is based on a multiple Lyapunov function that depends on the modes of both switching levels. Investigating the evolvement of the Lyapunov function leads to a sufficient condition for the stability analysis of the system. Correspondingly, the existence of a stabilizing controller is ensured, and the relevant state-feedback gains are designed such that the controlled system is stochastically stable in the second mean sense. In the overall analysis and synthesis procedure, time-varying norm-bounded uncertainties in dynamics are also taken into account to provide general results. Notably, the sufficient conditions obtained by the proposed systematic approach are less conservative than the previous design methods [14,18,21–23]; the main reason is the ability of the present approach to consider the individual active TR matrix at each time point instead of all of the possible TR matrices. The other reason is its ability to model the effects of the imperfect representation of the system as additive uncertainties for each TR matrix and compensate it. It is also worth mentioning that, all of the conditions obtained by the proposed approach, as well as the relevant controller gains, appear in the form of a set of finite dimensional linear matrix inequalities and equalities, which can be solved easily with the existing optimization techniques.

The organization of the paper is as follows. In Section 2, the NHMJLS is introduced in detail. Then, a brief summary of the stochastic terms, definitions and useful lemmas are given, and, finally, the problem is formulated in this section. In Section 3, the main results are provided; the TR partitioning technique is first addressed, and then the associated switching Markovian structure is constructed and verified. Finally, the stability of the system is investigated, and the controller design problem is tackled. An illustrative example related to a single-machine infinite-bus power system is provided in Section 4 to demonstrate the efficiency and potential of the newly developed theoretical results. Comparative simulations to the well-known polytopic approach [14,18,21–23] for dealing with NHMJLSs are also included in this section. Finally, there are concluding remarks in Section 5.

2. Preliminaries and problem formulation

Consider the following non-homogeneous Markov jump linear system in the probability space (Ω, F, ρ), where Ω, F and ρ represent the sample space, the algebra of events and the probability measure on F, respectively.

$$\begin{cases} \dot{x}(t) = \hat{A}(r_t, t)x(t) + \hat{B}(r_t, t)u(t), \\ x(t_0) = x_0, \; r_{t_0} = r_0 \end{cases} \tag{1}$$

$x(t) \in \mathbb{R}^n$ is the state vector with x_0 as the initial state, and $u(t) \in \mathbb{R}^m$ is the controlled input vector; $\mathbb{R}^n$ and $\mathbb{R}^m$ denote the n- and m-dimensional Euclidean space. System matrices $\hat{A}(r_t, t)$ and $\hat{B}(r_t, t)$represent mode-dependent with compatible dimensions defined by

$$\hat{A}(r_t, t) = A(r_t) + \Delta A(r_t, t), \quad \hat{B}(r_t, t) = B(r_t) + \Delta B(r_t, t) \tag{2}$$

where $A(r_t)$ and $B(r_t)$ are known, mode-dependent system matrices. $\Delta A(r_t, t)$ and $\Delta B(r_t, t)$ are unknown, mode-dependent matrices representing time-varying, norm-bounded parametric uncertainties. They are supposed to have the following form

$$\Delta A(r_t, t) = D_A(r_t)F_A(r_t, t)E_A(r_t), \quad \Delta B(r_t, t) = D_B(r_t)F_B(r_t, t)E_B(r_t) \tag{3}$$

in which $D_A(r_t)$, $D_B(r_t)$, $E_A(r_t)$ and $E_B(r_t)$ are known matrices; and $F_A(r_t, t)$ and $F_B(r_t, t)$ are time-varying, unknown, measurable matrices satisfying $F_A^T(r_t, t)F_A(r_t, t) \le I$ and $F_B^T(r_t, t)F_B(r_t, t) \le I$respectively.

The process $\{r_t, t \ge 0\}$ is a continuous-time non-homogeneous Markov process taking values in the finite set $\underline{N} = \{1, 2, \ldots, N\}$ that describes switching between different modes. r_0 is the initial mode, and the time-varying transition rate matrix is defined by $\Lambda(t) = [\lambda_{ij}(t)], \; i, j = 1, 2, \ldots, N$ where

$$\Pr\{r_{t+h} = j \mid r_t = i\} = \begin{cases} \lambda_{ij}(t)h + o(h) & i \ne j \\ 1 + \lambda_{ii}(t)h + o(h) & i = j \end{cases} \; i, j \in \underline{N} \tag{4}$$

such that $\lim_{h \to 0}(o(h)/h) = 0$. $h > 0$ is the dwell time and $\lambda_{ij}(t) \ge 0$, denotes the instantly and unconditionally time-varying transition probability from mode i at time t to mode j at time $t+h$. TRs satisfy the condition $\lambda_{ii}(t) = -\sum_{j=1, j \ne i}^{N} \lambda_{ij}(t)$, meaning that the values are never negative and that, with probability one, it must move from state i to some state j.

In what follows, one definition and one lemma that will be used in the development of the main results are recalled.

Definition. [9] For any initial mode r_0 and any given initial state vector x_0, the uncertain system (1) with $u(t)=0$ is said to be robustly stochastically stable in the second mean sense if the following condition holds for all admissible uncertainties, where $E\{.\}$