ISA Transactions 65 (2016) 72–80

Contents lists available at ScienceDirect

ISA Transactions

journal homepage: www.elsevier.com/locate/isatrans

Research Article

Robust Takagi-Sugeno fuzzy control for fractional order hydro-turbine governing system

Bin Wang [a], Jianyi Xue [b], Fengjiao Wu [a], Delan Zhu [a,*]

[a] College of Water Resources and Architectural Engineering, Northwest A&F University, Yangling 712100, Shaanxi, PR China
[b] School of Electrical Engineering, Xi'an Jiaotong University, Xi'an 710049, Shaanxi, PR China

ARTICLE INFO

Article history:
Received 26 November 2015
Received in revised form
22 May 2016
Accepted 23 June 2016
Available online 18 July 2016

Keywords:
Hydro-turbine governing system
Fractional order
Robust T-S fuzzy control
Linear matrix inequality
Random disturbances

ABSTRACT

A robust fuzzy control method for fractional order hydro-turbine governing system (FOHGS) in the presence of random disturbances is investigated in this paper. Firstly, the mathematical model of FOHGS is introduced, and based on Takagi-Sugeno (T-S) fuzzy rules, the generalized T-S fuzzy model of FOHGS is presented. Secondly, based on fractional order Lyapunov stability theory, a novel T-S fuzzy control method is designed for the stability control of FOHGS. Thirdly, the relatively loose sufficient stability condition is acquired, which could be transformed into a group of linear matrix inequalities (LMIs) via Schur complement as well as the strict mathematical derivation is given. Furthermore, the control method could resist random disturbances, which shows the good robustness. Simulation results indicate the designed fractional order T-S fuzzy control scheme works well compared with the existing method.

1. Introduction

Recently, due to the wide application in mechanics [1], electrical engineering [2] and many other fields [3,4], fractional calculus has attracted many scholars' attention in various fields. Since many actual projects of engineering applications could be elegantly described by fractional order calculus, like fractional order permanent magnet synchronous motor (PMSM) system [5,6], chemical processing systems [7] and wind turbine generators [8], fractional calculus still has great potential especially for the description of hereditary and memory attributes of numerous processes and materials [9,10].

As more attention is paid to the sustainable generation of power, hydropower plays an increasingly important role in the world's energy strategy. Well known, hydro-turbine governing system (HGS) is essential important in a hydroelectric station, which running conditions would directly affect the stable operation of the hydropower station. So modeling, analysis and control of HGS have arisen many researchers' interests [11–13]. In recent years, many scholars try to establish the nonlinear model of HGS [14–16]. However, most of the models are on the basis of integer order calculus. As we all know, HGS is a highly coupling, nonlinear as well as non-minimum phase system. For this reason, integer calculus is not suitable for describing complex hydro-turbine governing system. According to the history-dependent and memory character of hydraulic servo system, the fractional order hydro-turbine governing system (FOHGS) that is more in line with actual project is considered in this paper.

Many studies have indicated that the hydro-turbine governing system exhibits nonlinear even chaotic vibration in non-rated operating conditions [17,18]. Besides, due to the randomness of the load, the hydro-turbine governing system would be affected by the random disturbance, which may also lead to the unstable operation. So it is very important to design robust controller for suppressing nonlinear even chaotic vibration of HGS.

Recently, fractional order nonlinear control has attracted increasing attention. Some control methods have been presented for stability control of fractional order nonlinear or chaotic systems, such as fuzzy control method, sliding mode control, pinning control and predictive control [19–22]. For fuzzy control field, the Takagi-Sugeno fuzzy control method is a classic control scheme [23]. The nonlinear model is expressed by fuzzy IF-THEN rules, and the certain region of the system state is locally represented by the linearization description. The total system is then an integration of these local linearization models. Recently, the fuzzy control method that is deemed as one of the effective control techniques has attracted more and more scholars' attention [24–26]. However, the traditional T-S fuzzy control scheme is also facing some challenges. Firstly, the traditional T-S fuzzy control has been mostly applied to integer order systems. Well known, the controllability

* Corresponding author.
E-mail addresses: binwang@nwsuaf.edu.cn (B. Wang), dlzhu222@126.com (D. Zhu).

http://dx.doi.org/10.1016/j.isatra.2016.06.014
0019-0578/

and stability of fractional order system differ from integer order one. Whether T-S fuzzy control scheme could be applied to fractional order nonlinear systems? There have been some results about the applicability of fuzzy control into fractional order system [27]. However, the control quality is not very good. Besides, the oscillation occurs more frequently before the states become stable. And the conventional fuzzy control method has little ability to resist external disturbances.

Motivated by the discussions above, the major contributions of this paper include the following points. Firstly, based on fractional order Lyapunov stability theory, a novel T-S fuzzy control method is designed for stability control of fractional order hydro-turbine governing system. Secondly, the more relaxed and simplified sufficient stability conditions are given as a new set of linear matrix inequalities, which have been guaranteed by strict mathematical derivation. Thirdly, the control method has good robustness, which could resist the random disturbances. Finally, simulation results have demonstrated the robustness and effectiveness of this new approach when compared with the existing one.

The remaining contents of our paper are organized as follows. In Section 2, the fractional calculus and T-S fuzzy model are introduced. In Section 3, mathematical model of fractional order hydro-turbine governing system is presented. The design of controller for FOHGS is presented in Section 4. Numerical simulations are drawn in Section 5. Section 6 concludes this paper.

2. Preliminaries

2.1. Fractional calculus

During all of the fractional calculus definitions, Riemann-Liouville and Caputo fractional order operators are most commonly used which are introduced as follows:

Definition 1. [28]: The q-th fractional order Riemann-Liouville integration of function $f(t)$ is defined by:

$$ {}_{t_0}I_t^q f(t) = {}_{t_0}D_t^{-q} f(t) = \frac{1}{\Gamma(q)} \int_{t_0}^{t} \frac{f(\tau)}{(t-\tau)^{1-q}} d\tau, \tag{1} $$

where $\Gamma(\cdot)$ is the Gamma function.

Definition 2. [28]: The Riemann-Liouville fractional derivative of order $q>0$ of a continuous function $f(t)$ is defined as the$n$th derivative of fractional integral (1) of order $n-q$:

$$ {}_{t\_0}^{RL}D_t^q f(t) = \left(\frac{d}{dt}\right)^n I^{n-q} f(t) = \frac{1}{\Gamma(n-q)} \frac{d^n}{dt^n} \int_{t_0}^{t} \frac{f(\tau)}{(t-\tau)^{q-n+1}} d\tau, \tag{2} $$

where n is the smallest integer larger than or equal to q, and $\Gamma(\cdot)$ denotes the Gamma function.

Definition 3. [28]: The Caputo fractional derivative of order $q>0$ of a continuous function $f(t)$ at time instant $t \ge 0$ is defined as the fractional integral (1) of order $n-q$ of the nth derivative of $f(t)$:

$$ {}_{t\_0}^{C}D_t^q f(t) = I^{n-q} \left(\frac{d}{dt}\right)^n f(t) = \frac{1}{\Gamma(n-q)} \int_{t_0}^{t} \frac{f^n(\tau)}{(t-\tau)^{q-n+1}} d\tau, \tag{3} $$

where n is the smallest integer number larger than or equal to q, and $\Gamma(\cdot)$ denotes the Gamma function.

The Caputo fractional derivative has a popular application in engineering, which is adopted in this paper.

2.2. Generalized T-S fuzzy model

When it comes to fractional order nonlinear system, the generalized T-S fuzzy model is widely applied to construct the system model, which is on the basis of integer one. The fractional order generalized T-S fuzzy model could be represented via superposition of a set of fractional order sub-linear models, which is given as:

Rule R^i: IF $z_1(t)$ is M_{i1} and $\cdots$ and $z_n(t)$ is M_{in}

$$ \text{THEN} \quad \frac{d^q x}{dt^q} = A_i x(t) + B_i \tilde{u(t)} + G_i w(t) \quad (i=1,2,\cdots,r) \tag{4} $$

where $A_i \in R^{n\times n}$, the fuzzy set is $M_{ij} (j=1,2,\cdots,n)$ and the IF-THEN rules number is r, the state vector is $x(t) \in R^n$, the premise variables are $z(t)=[z_1(t), z_2(t), \cdots, z_n(t)]$ and $q(0<q\le 1)$ are the fractional orders, the control input is $\tilde{u(t)}$, the external disturbance is $w(t)$, $G_i \in R^{n\times n}$ is the coefficient matrix of the external disturbance. The overall output is expressed as:

$$ \frac{d^q x}{dt^q} = \frac{\sum_{i=1}^{r} \omega_i(z(t)) A_i x(t)}{\sum_{i=1}^{r} \omega_i(z(t))} + \frac{\sum_{i=1}^{r} \omega_i(z(t)) B_i \tilde{u(t)}}{\sum_{i=1}^{r} \omega_i(z(t))} + \frac{\sum_{i=1}^{r} \omega_i(z(t)) G_i w(t)}{\sum_{i=1}^{r} \omega_i(z(t))} \tag{5} $$

where $\omega_i(z(t)) = \prod_{j=1}^{n} M_{ij}(z_j(t))$, $M_{ij}(z_j(t))$ is the grade of membership of $z_j(t)$ in M_{ij}, meeting the following expression

$$ \begin{cases} \sum_{i=1}^{r} \omega_i(z(t)) > 0 \\ \omega_i(z(t)) \ge 0 \ \ (i=1,2,\ldots,r) \end{cases} \tag{6} $$

By introducing $h_i(z(t)) = \frac{\omega_i(z(t))}{\sum_{i=1}^{r} \omega_i(z(t))}$, the expression (5) is written as

$$ \frac{d^q x}{dt^q} = \sum_{i=1}^{r} h_i(z(t))[A_i x(t) + B_i \tilde{u(t)} + G_i w(t)](i=1,2,\cdots,r) \tag{7} $$

Note that

$$ \begin{cases} \sum_{i=1}^{r} h_i(z(t)) = 1 \\ h_i(z(t)) \ge 0 \ \ (i=1,2,\ldots,r) \end{cases} \tag{8} $$

where $h_i(z(t))$ represents IF-THEN rule normalized weight.

2.3. Parallel distributed compensation (PDC) control law

Based on PDC theory, the new fuzzy controller can be obtained as follows:

Rule R^i: IF $z_1(t)$ is M_{i1} and $\cdots$ and $z_n(t)$ is M_{in}

$$ \text{THEN} \quad \tilde{u(t)} = K_i x(t) \quad (i=1,2,\cdots,r) \tag{9} $$

where $K_i (i=1,2,\cdots,r)$ is the controller gain.

The overall controller could be written as

$$ \tilde{u(t)} = \sum_{i=1}^{r} h_i(z(t)) K_i x(t) \quad (i=1,2,\cdots,r) \tag{10} $$

Substituting (10) to (7), one gets

$$ \frac{d^q x}{dt^q} = \sum_{i=1}^{r} h_i(z(t)) A_i x(t) + \sum_{i=1}^{r} h_i(z(t)) \left[B_i \sum_{j=1}^{r} h_j(z(t)) K_j x(t) + G_i w(t) \right] \tag{11} $$

To simplify (11), making the following variants:

$$ \sum_{i=1}^{r} h_i A_i = h_1 A_1 + h_2 A_2 + \cdots + h_r A_r \tag{12} $$

$$ \sum_{i=1}^{r} h_i G_i = h_1 G_1 + h_2 G_2 + \cdots + h_r G_r \tag{13} $$