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Research Article

Function projective synchronization of complex networks with asymmetric coupling via adaptive and pinning feedback control

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ABSTRACT

The problem on function projective synchronization (FPS) is investigated for complex networks via hybrid control. In contrast to existing works, the asymmetric coupling matrix was considered. Based on adaptive and pinning feedback control methods, new FPS criteria are proposed. Finally, three examples are provided to illustrate the effectiveness of the proposed methods.

1. Introduction

Complex networks [1,2] lie everywhere in our daily life, such as the Internet, world wide web, communication networks, social networks, genetic regulatory networks, power grid networks, and so on. In 1990, controlling chaos was proposed and shown that one can convert a chaotic attractor into any one of a large number of possible attracting time-periodic motions by making only small time-dependent perturbations of an available system parameter [1]. Synchronization of complex networks has become an active field of researches [3–6], because the synchronization mechanism can explain well many natural phenomena, including the synchronous information exchange in the Internet and world wide web, and the synchronous transfer of digital or analog signals in communication networks. In 1998, focusing on the transition from a regular lattice to a random graph, Watts and Strogatz introduced an interesting model, i.e., the small-world network [4].

From the control strategy point of view, all kinds of control approaches have been proposed in the field of the synchronization of complex networks. These approaches can be divided into two classes in general. On the one hand, it is based on the continuous updating feedback signals, for instance, time-delay feedback control [7], adaptive control [8,9], nonlinear feedback control [10,11]. On the other hand, it is based on the discrete signals updated at instant times, such as sampled-data control [12,13], impulsive control [14,15]. The traditional method to synchronize a complex network is to add a controller to each of the network nodes to tame the dynamics to approach a desired synchronization trajectory. However, a complex network is normally composed of a large number of high-dimensional nodes, and it is expensive and infeasible to control all the neurons. Motivated by this practical consideration, the idea of controlling a small portion of nodes, named pinning control, was introduced in [16], and many pinning algorithms have been reported for the synchronization of dynamical networks (see [17]). Obviously, the pinning control method reduces the control cost to a certain extent by reducing the amount of controllers added to the nodes.

Up to now, all sorts of different synchronization phenomena, such as complete synchronization (CS) [18], generalized synchronization (GS) [19], phase synchronization [20], anticipated synchronization [21], projective synchronization (PS) [22], etc., have been reported in the literature. CS means that the coupled chaotic systems remain in step with each other in the course of time. Only in coupled systems with identical elements (i.e., each component having the same dynamics and parameter set), can we observe CS. PS is the dynamical behavior in which the amplitude of the driver state variable and that of the response synchronize up to a constant scaling factor (a proportional relation). Recently, a new type of synchronization phenomenon, called function projective synchronization (FPS), has been proposed and extensively studied

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[23–29]. FPS is the more general definition of chaotic synchronization. Let the scaling function be unity or constant, one can obtain complete synchronization or projective synchronization. Amongst all kinds of chaos synchronization, projective synchronization is one of the most interesting problems and has been especially extensively studied during recent years because of the proportionality between its synchronized dynamical states, which can be used to extend binary digital to M-nary digital communication for achieving fast communication [23,24]. FPS has attracted the interest of many researchers in various fields. FPS means that the driver and response systems could be synchronized up to a scaling function [25,26]. Further results on FPS of Genesio–Tesi chaotic systems were reported in [27]. FPS on dynamical networks has been reported by Zhang et al. in [28], where FPS with the desired scaling function is realized in drive-response dynamical networks with coupled partially linear chaotic nodes. In [31], based on the contradiction method and analysis technique, the exponential function projective synchronization of impulsive neural networks with mixed time-varying delays was investigated. Wu and Lu [32] investigated generalized function projective (lag, anticipated and complete) synchronization between two different complex networks with nonidentical nodes. The work in [33] studied the problem of FPS for general complex dynamical networks with time delay. Wu et al. in [34] discussed the generalized matrix projective synchronization problem of general colored networks with different-dimensional node dynamics. However, to the best of our knowledge, there are very few or even no results on the FPS of complex networks with asymmetric coupling matrix, while the FPS plays a key role in secure communication and other engineering fields. How one realizes FPS in complex networks with asymmetric coupling is still an open and challenging problem. Therefore, it is a crucial to investigate complex networks subject to asymmetric coupling. In this paper, the FPS of general complex networks will be investigated, which means that the nodes of complex networks could be synchronized up to an equilibrium point or periodic orbit with a desired scaling function.

Motivated by the above discussions, in this paper, we will study the FPS for complex dynamical networks with asymmetric coupling. Firstly, FPS is investigated via adaptive feedback control. Secondly, FPS is studied via pinning control with adaptive coupling strength. In contrast to previous results, it is not necessary for the coupling matrix to satisfy symmetric or nonnegative criteria. Thirdly, we decompose asymmetric matrix into symmetric, antisymmetric and diagonal matrices. By using some sufficient methods to deal with coupling matrix and by employing Lyapunov function, some sufficient conditions for synchronization of complex networks with asymmetric coupling are derived. Finally, three numerical examples are given to illustrate the effectiveness of the proposed methods. The key contributions made in this paper can be summarized as follows: (1) they assumed coupling matrix to be symmetric and off-diagonal entry to be nonnegative in [31–35]. However, we can reduce constraint about coupling matrix; (2) the nonlinear controllers are very complicated and difficult to implement in [31]; (3) they only considered linear feedback controller, but we can investigate other control strategies to achieve synchronization of complex networks.

The remainder of the paper is organized as follows. The network model is introduced and some necessary lemmas are given in Section 2. Section 3 discusses FPS of the complex dynamical networks with asymmetric coupling by the hybrid control method with nonlinear and adaptive linear feedback controller. Section 4 discusses FPS of the complex dynamical networks with asymmetric coupling by the hybrid control method with pinning coupling controller. Corresponding criteria for guaranteeing FPS are obtained. The theoretical results are verified numerically by two representative examples in Section 4. Finally, this paper is concluded in Section 5.

Notation: Throughout this paper, $\Re^n$ denotes n-dimensional Euclidean space and $\Re^{n\times n}$ is the set of all $n\times n$ real matrices. For symmetric matrices X and Y, the notation $X>Y$ $(X\geq Y)$ means that the matrix $X-Y$ is positive definite (nonnegative).

2. Preliminaries

Consider a complex dynamical network consisting of N identical nodes with asymmetric coupling:

$$\dot{x}_i(t)=f(x_i(t))+\sum_{j=1}^{N} g_{ij}x_j(t)+u_i(t), \tag{1}$$

where $i=1,2,\ldots,N$, $x_i(t)=(x_{i1}(t),\ldots,x_{in}(t))^T\in\Re^n$ is the state vector of node i, $f:\Re^n\to\Re^n$ is continuously differentiable. $u_i(t)$ is the control input. $G=(g_{ij})_{N\times N}$ is the coupling matrix satisfying $\sum_{j=1}^{N} g_{ij}=0$. The off-diagonal elements of G are not assumed to be nonnegative.

Definition 2.1. The dynamical networks (1) are said to achieve FPS if there exists a continuously differentiable scaling function $\alpha(t)$ such that

$$\lim_{t\to\infty}\|e_i(t)\|=\lim_{t\to\infty}\|x_i(t)-\alpha(t)s(t)\|=0,\quad i=1,2,\ldots,N,$$

where $\|\cdot\|$ stands for the Euclidean vector norm and $s(t)\in\Re^n$ is a solution of an isolate node, satisfying $\dot{s}(t)=f(s(t))$.

Remark 2.1. If the scaling function $\alpha(t)=1$ or $\alpha(t)=-1$, then the synchronization problem will be reduced to the complete synchronization or anti-synchronization. If the scaling function $\alpha(t)=0$, then the synchronization problem will be turned into a chaos control problem.

Define the error vectors as

$$e_i(t)=x_i(t)-\alpha(t)s(t)\quad (i=1,2,\ldots,N). \tag{2}$$

Then, the following error dynamical network can be obtained:

$$\dot{e}_i(t)=f(x_i(t))-\alpha(t)f(s(t))+\sum_{j=1}^{N} g_{ij}e_j(t)-\dot{\alpha}(t)s(t)+u_i(t). \tag{3}$$

In this paper, the asymmetric coupling matrix G can be decomposed into three matrices

$$G=\overline{G}+\tilde{G}+\hat{G}, \tag{4}$$

where $\overline{G}$ is a symmetric and zero row sum matrix,

$$\overline{G}=[\overline{g}_{ij}]:\begin{cases}\overline{g}_{ij}=(g_{ij}+g_{ji})/2, & i\neq j;\\ \overline{g}_{ij}=-\sum\limits_{k=1,k\neq i}^{N}\overline{g}_{ik}, & i=j,\end{cases}$$

$\tilde{G}$ is an antisymmetric matrix,

$$\tilde{G}=[\tilde{g}_{ij}]:\begin{cases}\tilde{g}_{ij}=(g_{ij}-g_{ji})/2, & i\neq j;\\ \tilde{g}_{ij}=0, & i=j,\end{cases}$$

$\hat{G}$ is a diagonal matrix,

$$\hat{G}=[\hat{g}_{ij}]:\begin{cases}\hat{g}_{ij}=0, & i\neq j;\\ \hat{g}_{ij}=-\sum\limits_{k=1,k\neq i}^{N}\tilde{g}_{ik}, & i=j.\end{cases}$$

Remark 2.2. Due to coupling matrix G being asymmetric, we cannot guarantee the derivative of the Lyapunov function to be nonnegative, so we cannot achieve synchronization via stability theory. By decomposing asymmetric coupling matrix G into

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