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Finite-time synchronization for second-order nonlinear multi-agent system via pinning exponent sliding mode control

Huazhou Hou, Qingling Zhang *

Institute of System Science, Northeastern University, Shenyang, Liaoning Province 110819, China

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ABSTRACT

In this paper we investigate the finite-time synchronization for second-order multi-agent system via pinning exponent sliding mode control. Firstly, for the nonlinear multi-agent system, differential mean value theorem is employed to transfer the nonlinear system into linear system, then, by pinning only one node in the system with novel exponent sliding mode control, we can achieve synchronization in finite time. Secondly, considering the 3-DOF helicopter system with nonlinear dynamics and disturbances, the novel exponent sliding mode control protocol is applied to only one node to achieve the synchronization. Finally, the simulation results show the effectiveness and the advantages of the proposed method.

1. Introduction

In the past decades, multi-agent systems have attracted more and more attention, fruitful results have been achieved concerned with the multi-agent systems. For example, in [1], the authors considered the consensus problems with H_∞ and weighted H_∞ bounds for a homogenous team of linear time invariant (LTI) multi-agent systems with a switching topology and directed communication network graph, and finite-time consensus problem for second-order multi-agent systems with disturbances via integral sliding mode control has been considered in [2]. Paper [3] investigated the discrete-time double-integrator consensus of multi-agent systems with input constraints and directed switching proximity topologies by using model predictive control approach. In [4], authors studied the output synchronization of discrete-time multi-agent systems with directed communication topologies which contains a spanning tree and exosystem as its root. Ref. [5] investigated the synchronization problem for discrete-time linear multi-agent systems with time-varying network topology and proposed the synchronization reachable topology. Among various research interests, one particular area of interest is finite-time consensus. Compared with asymptotic consensus, meaning that the consensus can be achieved asymptotically, finite-time consensus has numerous benefits, such as disturbance rejection property and robustness against uncertainties. There are many results relating to finite-time consensus, for example, in [6], the authors considered the finite-time consensus tracking control of multirobot systems by the terminal sliding mode control, and further extended the results to the multirobot systems with disturbances. Ref. [7] investigated the distributed finite-time consensus problem of networked agents with second-order integrators, a saturated protocol is proposed based on both relative position and relative velocity measurements. In [8], the authors considered the distributed finite-time consensus problem of second-order multi-agent systems in the presence of bounded disturbances via discontinuous and continuous integral sliding mode protocols. Ref. [9] analyzed the finite-time convergence of a nonlinear but continuous consensus algorithm for multi-agent networks with unknown inherent nonlinear dynamics and proposed a novel comparison based tool to guarantee finite-time convergence. However, the aforementioned results are all concerned with the finite-time consensus, the results related to finite-time synchronization in multi-agent systems are comparatively rare, thus finite-time synchronization in multi-agent systems still needs to be considered since synchronization is a significantly important phenomenon in physical systems [10–13].

In many cases, the multi-agent system may contain complex nonlinear dynamics and disturbances, thus the multi-agent system may not achieve synchronization themselves, then, some

* Corresponding author.
E-mail address: qlzhang@mail.neu.edu.cn (Q. Zhang).

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controllers should be designed to drive the multi-agent system to the synchronization state, moreover, even the multi-agent system can achieve synchronization state by its intrinsic structure, the final synchronization state may not be the expected ones, hence, the extra control may be needed to adjust the synchronized state with the aforementioned factors such as nonlinear dynamics and disturbances. In such context, several control schemes for multi-agent synchronization can be found in the literature, such as optimal distributed control [14], adaptive feedback control [15], harmonic control [16] and so forth. Among these control protocols, sliding mode control seems to be one of the most effective approaches to cope with the uncertainties or the disturbances. Sliding mode control is a nonlinear control method which alters the dynamics of a system by using a discontinuous control signal and forces the system to slide along a prescribed switching manifold. Compared with other control methods, sliding mode control has attracted a significant interest due to its simplicity, high robustness to external disturbances and low sensitivity to the system parameter variations [17,18]. Sliding mode control method has also been used in the multi-agent systems to achieve control objective, for example, [19] discussed the asymptotic consensus problem and finite-time leader-following consensus problem of second-order nonlinear multi-agent systems with directed communication topology via terminal sliding mode control. In [20], the authors proposed sliding mode observer based robust fault estimation method to solve the fault estimation in linear multi-agent networks.

In this paper, we investigate the finite-time synchronization for second-order multi-agent systems and 3-DOF helicopter system via pinning exponent sliding mode control. In second-order multi-agent system and 3-DOF helicopter system, we consider the nonlinear dynamics and disturbances factors, which are more practical, by pinning only one node in the system with proper exponent sliding mode controller. We derive the condition that ensures the finite-time synchronization. The highlights of this paper are listed as follows. Firstly, for the multi-agent systems, if the system cannot synchronize by its intrinsic structure, the synchronization can be achieved by pinning only one node in the system with the designed exponent sliding mode controller, and compared with the method in the references, our method is faster. It should be noted that the pinning scheme makes sense in practical situations because control with pinning scheme can decrease the cost of the system and it becomes easier to locate the fault when the control systems break down. Secondly, if the disturbances are considered in the system, the synchronization can also be achieved by our robust pinning sliding mode protocol. Thirdly, the proposed method can be applied to the practical 3-DOF helicopter system and achieve the control target.

The rest of this paper is organized as follows: in Section 2, we give out the preliminaries and problem statement. In Section 3 we design the pinning exponent sliding mode controller for the multi-agent system to solve the synchronization problem and in Section 3.1 we demonstrated that how the change of parameters in sliding surface can affect the response of the system. In Section Section 4 we apply the pinning exponent sliding mode controller to the 3-DOF helicopter system. Section 5 presents some simulation examples to verify the effectiveness of the proposed method. Finally, some conclusions are drawn in Section 6.

2. Background and preliminaries

2.1. Graph theory

For the considered second-order multi-agent systems, assume that each agent is a node and the information exchange between n agents can be denoted by an undirected graph $G=\{V,E,A\}$. $V=\{\nu_i, i=1,\dots,n\}$ is the set of vertices, $E\subseteq V\times V$ is the set of edges and $A=[a_{ij}]\in R^{n\times n}$ is the weighted adjacency matrix of the graph G. The node indexes belong to a finite index set $\Gamma=\{1,\dots,n\}$. If there is an edge between agent i and agent j, i.e., $(\nu_i,\nu_j)\in E$, then, $a_{ij}=a_{ji}>0$. If there is not any edge between agent i and agent j, then $a_{ij}=a_{ji}=0$. Moreover, assume that $a_{ii}=0$ for all $i\in\Gamma$. The set of neighbors of node ν_i is denoted by $N_i=\{j:(\nu_i,\nu_j)\in E\}$.

The Laplacian matrix of undirected graph G is $L=[l_{ij}]\in R^{n\times n}$, where $l_{ii}=\sum_{j=1}^{n}a_{ij}$ and $l_{ij}=-a_{ij},\ i\neq j$. A path in the graph G represents a way to obtain from an origin node ν_i to a destination node ν_j by traversing edges in the graph. An undirected graph G is connected if there is a path between any pair of distinct nodes. The connection weight the ith agent and the leader is denoted by d_i with $d_i>0$ if there is an edge between the ith agent and the leader. For the direct multi-agent system, if there is an edge from agent j to agent i, then it is said that node ν_j can reach node ν_i and $a_{ij}>0$ is the weight associated with the edge e_{ij}; otherwise, $a_{ij}=0$.

2.2. Some useful lemmas

Lemma 1 ([2]). *Consider the following system*

$$\dot{x}=f(x,t),\quad f(0,t)=0,\ x\in U_0\subset R^n \tag{1}$$

Suppose that there is a continuous differentiable positive definite function $V(x)$ defined in a neighborhood of the origin, and real numbers $c>0$, $\alpha\in(0,1)$, such that $\dot{V}\le -cV^{\alpha}$, then the origin of the system is finite-time stable, and the upper bound of the settling time is satisfied with

$$T\le\frac{V^{1-\alpha}(0)}{c(1-\alpha)} \tag{2}$$

Lemma 2 ([20]). *For the multi-input system $\dot{x}(t)=Ax(t)+Bu(t)$, if we can design a feedback control $u(t)=K_1x(t)+e_i\nu(t)$ to transfer the multi-input system into a single-input system $\dot{x}(t)=(A+BK_1)x(t)+b_i\nu(t)$, where $K_1\in R^{m\times n}$ is a feedback gain matrix; $e_i^T=[0,0,\dots,0,\underbrace{1}_{i},0,\dots,0,0]$, $\nu(t)$ is a new single input. The controllability of original multi-input system will not be changed if we can find a proper feedback gain matrix K_1, which means the controllability of single-input system is equivalent to the original multi-input system.*

Lemma 3 ([21]). *The following matrix inequality:*

$$\begin{bmatrix} Q(x) & \Pi(x) \\ \Pi(x)^T & R(x) \end{bmatrix}>0$$

where $\Pi(x)$ depends affinity on x is equivalent to

$$R(x)>0,\quad Q(x)-\Pi(x)R(x)^{-1}\Pi(x)^T>0$$

Lemma 4 ([21]). *Let X,Y,F be real matrices of appropriate dimensions with $F^T(t)F(t)\le I$ then*

$$XFY+Y^TF^TX^T\le XX^T+Y^TY$$

2.3. Problem formulation

In this paper, we consider a class of multi-agent system composed of n second-order agents. The agents exchange the state information through communication networks. The dynamic of the agent is described as follows:

$$\begin{cases}\dot{x}_i(t)=v_i(t)\\ \dot{v}_i(t)=f(t,x_i(t),v_i(t))+u_i(t)+g_i(t)\end{cases} \tag{3}$$

where $i=1,\dots,n$, $x_i(t)$ and $v_i(t)$ are the position and velocity states of agent i, respectively. $u_i(t)$ is the sliding mode control to be