ARTICLE IN PRESS

ISA Transactions ∎ (∎∎∎∎) ∎∎∎–∎∎∎

Contents lists available at ScienceDirect

ISA Transactions

journal homepage: www.elsevier.com/locate/isatrans

Singularity-free backstepping controller for model helicopters

Yao Zou [a,b], Wei Huo [a,b,*]

[a] The Seventh Research Division, Beihang University, Beijing 100191, China
[b] School of Automation Science and Electrical Engineering, Beihang University, Beijing 100191, China

ARTICLE INFO

Article history:
Received 6 December 2015
Received in revised form
5 June 2016
Accepted 20 June 2016
This paper was recommended for publication by Dr. Y. Chen.

Keywords:
Model helicopter
Trajectory tracking
Singularity-free controller
Backstepping technique
Auxiliary dynamic system

ABSTRACT

This paper develops a backstepping controller for model helicopters to achieve trajectory tracking without singularity, which occurs in the attitude representation when the roll or pitch reaches $\pm\frac{\pi}{2}$. Based on a simplified model with unmodeled dynamics, backstepping technique is introduced to exploit the controller and hyperbolic tangent functions are utilized to compensate the unmodeled dynamics. Firstly, a position loop controller is designed for the position tracking, where an auxiliary dynamic system with suitable parameters is introduced to warrant the singularity-free requirement for the extracted command attitude. Then, a novel attitude loop controller is proposed to obviate singularity. It is demonstrated that, based on the established criteria for selecting controller parameters and desired trajectories, the proposed controller realizes the singularity-free trajectory tracking of the model helicopter. Simulations confirm the theoretical results.

1. Introduction

In recent decades, model helicopters have attracted much attention due to their capacities of vertically taking-off and landing, hovering and low-speed flight. They have been widely utilized in reconnaissance, surveillance and other stationary flight missions [1,2]. Since model helicopter dynamics are featured with under-actuation, aerodynamic complexity and strong coupling, the autonomous control of a model helicopter is still a challenge for researchers.

Since the model helicopter is an under-actuated system, whose 3-dof (degree of freedom) translational motions and 3-dof rotational motions are driven by four controls, command attitude extraction is a necessary step for controller designs. Generally, an inner-outer loop control structure is used to design controllers for the model helicopter [3]: firstly, a position loop controller is synthesized for the position tracking; then, a main rotor thrust (as one control) and a command attitude are extracted from the position loop controller; and finally, an attitude loop controller (including three controls) is exploited for the attitude tracking to the command attitude. Nevertheless, under this control structure, when the attitude is represented with Euler angles, the inner attitude loop controller cannot offset the error from the outer position loop due to the nonlinear command attitude extraction. In this case, the time-scale separation assumption is available [3], which requires a faster convergence of the attitude loop than the position loop. Under the time-scale separation assumption, various control approaches have been used for the position and attitude loop controller developments, such as nonlinear H_∞ with model predictive control [4], feedback linearization [5–7], sliding mode control [8], PID with model inversion blocks [9]. However, with the time-scale separation assumption, the stable position tracking is on the premise of the stable attitude tracking.

Backstepping technique is available for the controller development without the time-scale separation assumption [10,11]. Based on the backstepping strategy, He et al. [12] developed a controller to achieve the bounded trajectory tracking of a model helicopter. Lee et al. [13,14] combined the integral backstepping and dynamic extension to improve the tracking performance. The controllers in [12–14] are valid under the condition that the roll and pitch of the helicopter lie in $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$, but they fail to ensure the condition. Further, to obviate the nonlinear command attitude extraction, Raptis et al. [15] proposed a new attitude representation instead of Euler angles. Nevertheless, transformation between the Euler angles and the new attitude representation appears singularity when the roll or pitch of the helicopter reaches $\pm\frac{\pi}{2}$. Although the backstepping controller in [15] achieves the nonsingular exponentially stable trajectory tracking of the model helicopter, it will be invalid once the unmolded dynamics of the helicopter are taken into account. Using the attitude representation proposed in [15], the backstepping controllers in [16–18] are

* Corresponding author.
E-mail addresses: zouyao20@126.com (Y. Zou), weihuo@buaa.edu.cn (W. Huo).

http://dx.doi.org/10.1016/j.isatra.2016.06.010
0019-0578/

based on the hypothesis that no singularity occurs, but they cannot obviate it.

Moreover, the unmodeled dynamics of the model helicopter may affect the tracking performance, thus, they should be compensated. Neural networks [7,19], nonlinear damping [17] and disturbance observers [12,20] have been applied to compensate the unmodeled dynamics. However, the neural network may increase computational complexity [21], the nonlinear damping may introduce a large undesired control input [22], and the disturbance observer may bring in additional intricate dynamics [23]. With the prior knowledge of the upper bounds of the unmodeled dynamics, the most effective way is to directly compensate them with sign functions and their upper bounds [24]. However, when the sign function appears in the position loop controller, chattering phenomenon may occur in the extracted command attitude, which is unbeneficial for the attitude tracking. Although the sliding-mode observers proposed in [25–27] can relieve chattering, they do not establish a parameterized relation between the chattering attenuation and the tracking performance.

With the attitude representation proposed in [15], this paper develops a singularity-free trajectory-tracking controller for the model helicopter. The helicopter system is firstly simplified into a cascaded structure with unmodeled dynamics. Based on the simplified model, the backstepping technique is used to exploit the controller and the hyperbolic tangent function is applied to compensate the unmodeled dynamics. During the backstepping procedure, to avoid the aforementioned singularity in the attitude representation, it is necessary to impose the same non-singular requirement on the extracted command attitude. An auxiliary dynamic system with appropriate parameters is introduced to guarantee such non-singular requirement. It is proven that, with suitable controller parameters and desired trajectories, the proposed controller achieves the non-singular trajectory tracking of the model helicopter. Main contributions are listed as follows:

(i) during the position loop controller design, an auxiliary dynamic system is introduced, and criteria for choosing parameters are established to ensure the stability of it and the non-singular requirement of the extracted command attitude;
(ii) a singularity-free attitude controller with appropriate parameters and desired trajectory constraint is put forward;
(iii) instead of the sign function, the hyperbolic tangent function is introduced to compensate the unmodeled dynamics, and the parameterized relation between the chattering attenuation and the tracking performance is built.

The following sections are organized as follows: some mathematical preliminaries are presented in Section 2; control problems are stated in Section 3, controller design and stability analysis are provided in Section 4; Simulations are carried out in Section 5; and conclusions are drawn in Section 6.

2. Preliminaries

In this paper, $|\cdot|$ denotes the absolute value of a scalar, $\|\cdot\|$ denotes the Euclidean norm of a vector, S_x and C_x with $x\in\mathbb{R}$ are short for the trigonometric functions $\sin(x)$ and $\cos(x)$, $\overline{\lambda}(\cdot)$ and $\underline{\lambda}(\cdot)$ denote the maximum and minimum eigenvalues of a square matrix, $\boldsymbol{e_1}=[1,0,0]^T$, $\boldsymbol{e_2}=[0,1,0]^T$ and $\boldsymbol{e_3}=[0,0,1]^T$ denote three unit vectors. For $\boldsymbol{x}=[x_1,x_2,x_3]^T\in\mathbb{R}^3$, the superscript $\times$ denotes the transformation from $\boldsymbol{x}$ to a skew-symmetric matrix, namely,

$$\boldsymbol{x}^\times=\begin{bmatrix}0 & -x_3 & x_2\\ x_3 & 0 & -x_1\\ -x_2 & x_1 & 0\end{bmatrix}.$$

For $x\in\mathbb{R}$, define the hyperbolic tangent function $\tanh(x)=\frac{e^x-e^{-x}}{e^x+e^{-x}}$, which satisfies $|\tanh(x)|<1$ and $0<\frac{\tanh(x)}{x}<1$. For $\boldsymbol{x}=[x_1,\ldots,x_n]^T\in\mathbb{R}^n$, define the hyperbolic tangent function vector $\mathbf{tanh}(\boldsymbol{x})=[\tanh(x_1),\ldots,\tanh(x_n)]^T$.

Lemma 1 ([28]). *Suppose that $\boldsymbol{h}(\boldsymbol{x}):\mathcal{D}\to\mathbb{R}^n$ is a smooth function defined on $\mathcal{D}\subset\mathbb{R}^n$, and the Jacobian matrix of $\boldsymbol{h}$ is nonsingular at a point $\boldsymbol{x}=\boldsymbol{x_0}$, then on a suitable open subset $\mathcal{D}_0$ of $\mathcal{D}$, containing $\boldsymbol{x_0}$, $\boldsymbol{h}$ defines a local diffeomorphism.*

Lemma 2 ([18,29]). *Given $\epsilon>0$, the following inequality holds for $x\in\mathbb{R}$:*

$$0\le|x|-x\tanh\left(\frac{x}{\epsilon}\right)\le k_q\epsilon, \tag{1}$$

where k_q satisfies $k_q=e^{-(k_q+1)}$, i.e., k_q=0.2785.

Lemma 3 ([30,31]). *The following inequality holds for $x\in\{x\in\mathbb{R}||x|<k\}$:*

$$\ln\frac{k^2}{k^2-x^2}\le\frac{x^2}{k^2-x^2}. \tag{2}$$

Lemma 4. *Consider the system*

$$\ddot{\boldsymbol{\xi}}+\alpha\,\mathbf{tanh}(k\boldsymbol{\xi}+l\dot{\boldsymbol{\xi}})+\beta\,\mathbf{tanh}(l\dot{\boldsymbol{\xi}})-\boldsymbol{d}(t)=0, \tag{3}$$

where $\boldsymbol{\xi}\in\mathbb{R}^n$ and $\dot{\boldsymbol{\xi}}\in\mathbb{R}^n$ are states, $\boldsymbol{d}(t):\mathbb{R}^+\to\mathbb{R}^n$ is a disturbance, k,l,α and β are positive constants and satisfy

$$\sqrt{\frac{k}{l^2}\beta}\le\alpha\le\frac{1}{2}\left(\beta+\frac{k}{l^2}\right). \tag{4}$$

If there exist $\overline{d}>0$ and $\overline{t}>0$ such that

$$\|\boldsymbol{d}(t)\|<\overline{d}<\frac{k\beta}{2(l^2\beta+k)},\quad \forall t\ge\overline{t}. \tag{5}$$

then $\boldsymbol{\xi}$ and $\dot{\boldsymbol{\xi}}$ ultimately converge to the attractive set

$$\mathcal{Z}=\{[\boldsymbol{\xi}^T,\dot{\boldsymbol{\xi}}^T]^T\,|\,\|[k\boldsymbol{\xi}^T+l\dot{\boldsymbol{\xi}}^T,l\dot{\boldsymbol{\xi}}^T]^T\|<\overline{\mu}\}, \tag{6}$$

where $\overline{\mu}$ satisfies $\frac{l^2\beta+k}{k\beta}\overline{d}<\frac{\tanh^2(\overline{\mu})}{\overline{\mu}}<\frac{1}{2}$.

Proof. See Appendix A.

3. Problem statements

3.1. Helicopter model

In this paper, the helicopter is considered as a 6-dof rigid body. Let $\mathcal{I}=\{Oxyz\}$ denote an inertial frame whose origin O is located at a fixed point on the earth, and $\mathcal{B}=\{O_bx_by_bz_b\}$ denote a body frame whose origin O_b is located at the helicopter c.g. (center of gravity) (see Fig. 1). During the modeling, $\boldsymbol{p}=[p_x,p_y,p_z]^T$ and $\boldsymbol{v}=[v_x,v_y,v_z]^T$ respectively denote the position and velocity of the helicopter c.g. in $\mathcal{I}$, $\boldsymbol{\omega}=[\omega_x,\omega_y,\omega_z]^T$ denotes the angular velocity of the helicopter in $\mathcal{B}$, $\boldsymbol{\gamma}=[\phi,\theta,\psi]^T$ denotes the Euler angle vector (roll, pitch, yaw), and the corresponding rotation matrix $\boldsymbol{R}$ from $\mathcal{B}$ to $\mathcal{I}$, which is parametrized by $\boldsymbol{\gamma}$, is expressed as

$$\boldsymbol{R}=\begin{bmatrix}R_{11} & R_{12} & R_{13}\\ R_{21} & R_{22} & R_{23}\\ R_{31} & R_{32} & R_{33}\end{bmatrix}=\begin{bmatrix}C_\theta C_\psi & S_\phi S_\theta C_\psi-C_\phi S_\psi & C_\phi S_\theta C_\psi+S_\phi S_\psi\\ C_\theta S_\psi & S_\phi S_\theta S_\psi+C_\phi C_\psi & C_\phi S_\theta S_\psi-S_\phi C_\psi\\ -S_\theta & S_\phi C_\theta & C_\phi C_\theta\end{bmatrix}. \tag{7}$$

Referring to [32], the kinematic equations of the helicopter are described as

$$\dot{\boldsymbol{p}}=\boldsymbol{v}, \tag{8}$$

Please cite this article as: Zou Y, Huo W. Singularity-free backstepping controller for model helicopters. ISA Transactions (2016), http://dx.doi.org/10.1016/j.isatra.2016.06.010