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ISA Transactions ∎ (∎∎∎∎) ∎∎∎–∎∎∎

Contents lists available at ScienceDirect

ISA Transactions

journal homepage: www.elsevier.com/locate/isatrans

Research Article

Improved robustness and performance of discrete time sliding mode control systems☆

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ARTICLE INFO

Article history:
Received 7 October 2015
Received in revised form
10 June 2016
Accepted 5 August 2016
This paper was recommended for publication by Dr. Oscar Camacho.

Keywords:
Discrete time sliding mode control
Relative degree
Ultimate band
Robustness
Finite time convergence

ABSTRACT

This paper presents a theoretical analysis along with simulations to show that increased robustness can be achieved for discrete time sliding mode control systems by choosing the sliding variable, or the output, to be of relative degree two instead of relative degree one. In other words it successfully reduces the ultimate bound of the sliding variable compared to the ultimate bound for standard discrete time sliding mode control systems. It is also found out that for such a selection of relative degree two output of the discrete time system, the reduced order system during sliding becomes finite time stable in absence of disturbance. With disturbance, it becomes finite time ultimately bounded.

1. Introduction

Sliding mode control (SMC) has made its place in literature as a control which enables complete disturbance rejection in finite time, when the disturbance is bounded in nature and appears in the input channel. It works by applying a switching control, which brings the states of the system to a sliding surface in finite time, and once this sliding motion is achieved, the system is no longer affected by the disturbance. Though this idea is theoretically fascinating, it has practical limitations. The switching control needs actuators to switch at infinite frequency, which is not possible in the real world. Also, measurements by sensors and the control computation are done only after a specific time period. To remove this gap between theory and practice, discrete time sliding mode control (DSMC) theory was developed in [1–9]. Also many physical systems have inherently discrete time dynamics. It was seen that for these systems the states can no longer hit the sliding surface and stay there in presence of a disturbance, but converge inside an ultimate band around the surface in finite time. Hence the robustness of the system gets defined by the width of this ultimate band.

The discrete sliding mode control has traditionally been developed by taking outputs (or sliding variables) of relative degree one, i.e., the delay between the output and the control input is unit time step. This has given rise to proposals of various reaching laws of the form $s(k+1)=f(s(k))$, where $s(k)$ is the sliding variable at the kth time step. Since $s(k)$ is chosen as a relative degree one output, these reaching laws enable the calculation of control as $s(k+1)$, when calculated from the system dynamics, contains $u(k)$. The most well-known of these reaching laws are laid down in [5,3,2]. Of the above, the first paper deals with a switching reaching law and the other two with non-switching reaching laws. Even to this day, proposals of reaching laws are being laid down in literature, which exhibit different properties favourable to the design of control for a particular type of system. Some of these reaching laws are found in [10–17]. Very recent contributions highlighting the usefulness of DSMC can be found in [18,19].

However, all the above work proposes a sliding variable of relative degree one and finds out the ultimate band in each case. In the work presented in this paper, the authors aim to show that when the sliding variable is chosen with relative degree two, we get reduced width of the ultimate band and finite time stability during sliding in absence of disturbance. This is an important achievement, as this increases the robustness of the system as a whole, since robustness is directly related to the width of this ultimate band.

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http://dx.doi.org/10.1016/j.isatra.2016.08.006
0019-0578/

Please cite this article as: Chakrabarty S, Bartoszewicz A. Improved robustness and performance of discrete time sliding mode control systems. ISA Transactions (2016), http://dx.doi.org/10.1016/j.isatra.2016.08.006

A similar work has been recently presented in [20] where the reaching law given in [3] is used with an output of relative degree $r=n$, where n is the order of the system. The work is restrictive in the sense that r must be equal to n in order to obtain the properties of improved robustness and finite time stability. However, it has shown the path to the work in this paper where $r=2$ can bring about the desired properties of improved robustness and finite time stability for an $n \geq 2$ order system.

The reaching laws that are used in this paper are the ones given in [5,2]. The switching reaching law in [5] contains a proportional and a switching term along with a term containing a disturbance $d(k)$ (see Section 4). The non-switching reaching law in [2] contains a term which becomes zero in finite time in addition to this $d(k)$ (see Section 3). This disturbance term $d(k)$ is calculated as a direct functional relationship with the disturbance $f(k)$ in the actual system, such that the control input becomes devoid of any disturbance terms. In this paper, we shall see that for relative degree two output, the disturbance term $d(k)$ does not exhibit the same functional relationship with $f(k)$ as in the case of relative degree one output. It is easy to conclude that the bound of this term $d(k)$, which is denoted by d_m, will also change because of this. Since d_m has a direct influence on the ultimate bound, the ultimate bound will also vary accordingly. This observation laid to the work in this paper, which is described in detail in the following sections.

In the remainder of the work, the terms '*output*' and '*sliding variable*' will be used interchangeably. This is because traditionally relative degree is defined for the output of a system, and sliding variable is a constructed output, if not a measured one. The next section introduces the idea of relative degree of an output of a discrete linear time invariant (LTI) system, which is the system considered in this work. The property of finite time convergence of all states in the sliding mode for relative degree two output is also shown in this section. Sections 3 and 4 discuss the conditions required to achieve reduction in the width of the ultimate bound for a non-switching and a switching reaching law respectively. Section 5 shows simulation examples for both the non-switching and switching reaching laws, comparing the results with selection of relative degree one output. The conclusions of the paper are collected in Section 6.

Notations used in this paper: (1) $\|\bullet\|$ is the standard Euclidean 2-norm of a vector or a matrix, (2) $Ker(L)$ is the set of all vectors $x \in X$ which the linear transformation $L: x \mapsto y$ takes to $y = 0 \in Y$, X and Y being vector spaces, (3) $\det(A)$ denotes the determinant of the square matrix A.

2. Outputs with relative degree 1 and 2

Let us consider a discrete time LTI system

$$\begin{aligned} x_1(k+1) &= A_{11}x_1(k) + A_{12}x_2(k) \\ x_2(k+1) &= A_{21}x_1(k) + A_{22}x_2(k) + B_2u(k) + B_2f(k) \end{aligned} \tag{1}$$

where $x_1(k) \in \mathbb{R}^{(n-m)}$ and $x_2(k) \in \mathbb{R}^m$ are the n states and $u(k) \in \mathbb{R}^m$ is the input. The disturbance $f(k) \in \mathbb{R}^m$ is assumed to be bounded as $||f(k)|| \leq f_m$. The controls designed in the sequel guarantees bounded stability of the system in presence of such matched disturbance. The problem of unmatched disturbance has not been studied in this work.

The above makes $A_{11} \in \mathbb{R}^{(n-m)\times(n-m)}$, $A_{12} \in \mathbb{R}^{(n-m)\times m}$, $A_{21} \in \mathbb{R}^{m\times(n-m)}$, $A_{22} \in \mathbb{R}^{m\times m}$ and $B_2 \in \mathbb{R}^{m\times m}$. We assume $\det(B_2) \neq 0$. Obviously, written in the standard form of discrete LTI systems, we shall have

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ B_2 \end{bmatrix} \tag{2}$$

The working definition of relative degree of an output for a discrete time system is given below.

Definition 1. The relative degree r of an output $s(k)$ of a discrete time system is the order of delay of the output in which the input $u(k)$ first appears.

Hence, if the relative degree is r, then $s(k+i) = F_i(x(k)) \forall i < r$ and $s(k+i) = F_i(x(k), u(k), u(k+1), \ldots, u(k+i-r)) \forall i \geq r$.

2.1. Asymptotic stability with relative degree 1 output

For the above system, a relative degree one output is proposed as

$$s_1(k) = C_1^T x(k) = Cx_1(k) + I_m x_2(k) \tag{3}$$

where $C \in \mathbb{R}^{m\times(n-m)}$. Then

$$C_1^T B = \begin{bmatrix} C & I_m \end{bmatrix} \begin{bmatrix} 0 \\ B_2 \end{bmatrix} = B_2 \tag{4}$$

and we can calculate the control $u(k)$ from

$$s_1(k+1) = C_1^T Ax(k) + C_1^T Bu(k) + C_1^T Bf(k) \tag{5}$$

as obtained from the system dynamics, since B_2 is non-singular.

Design of C is done considering closed loop performance of the nominal system, i.e., when $f(k)=0$. Then the output hits zero in finite time and for relative degree one systems, we get $x_2(k) = -Cx_1(k)$. Hence, the closed loop system becomes

$$x_1(k+1) = (A_{11} - A_{12}C)x_1(k) \tag{6}$$

which is traditionally made asymptotically stable by choosing

$$\max(|\lambda_1|) < 1 \tag{7}$$

where λ_1 is an eigenvalue of $(A_{11} - A_{12}C)$. Since $x_2(k)$ is algebraically related to $x_1(k)$, it also settles down to zero asymptotically.

2.2. Finite time stability with relative degree 2 output

For the system (1), a relative degree two output will be

$$s_2(k) = C_2^T x(k) = Cx_1(k) \tag{8}$$

where $C \in \mathbb{R}^{m\times(n-m)}$ can be chosen same as in (3) or different, but satisfying the conditions in Theorem 1 below.

Now

$$C_2^T B = \begin{bmatrix} C & 0 \end{bmatrix} \begin{bmatrix} 0 \\ B_2 \end{bmatrix} = 0 \tag{9}$$

implying

$$s_2(k+1) = C_2^T Ax(k) + C_2^T Bu(k) + C_2^T Bf(k) = C_2^T Ax(k) \tag{10}$$

as calculated from the system dynamics (1) does not contain the control input $u(k)$. Then we need to assume

$$\begin{aligned} C_2^T AB &= \begin{bmatrix} C & 0 \end{bmatrix} \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} 0 \\ B_2 \end{bmatrix} \\ &= \begin{bmatrix} CA_{11} & CA_{12} \end{bmatrix} \begin{bmatrix} 0 \\ B_2 \end{bmatrix} \\ &= CA_{12}B_2 \end{aligned} \tag{11}$$

to be non-singular in order that the output is of relative degree 2,