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Research Article

Observer-based robust control of one-sided Lipschitz nonlinear systems

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ABSTRACT

This paper presents an observer-based controller design for the class of nonlinear systems with time-varying parametric uncertainties and norm-bounded disturbances. The design methodology, for the less conservative one-sided Lipschitz nonlinear systems, involves astute utilization of Young's inequality and several matrix decompositions. A sufficient condition for simultaneous extraction of observer and controller gains is stipulated by a numerically tractable set of convex optimization conditions. The constraints are handled by a nonlinear iterative cone-complementary linearization method in obtaining gain matrices. Further, an observer-based control technique for one-sided Lipschitz nonlinear systems, robust against L_2-norm-bounded perturbations, is contrived. The proposed methodology ensures robustness against parametric uncertainties and external perturbations. Simulation examples demonstrating the effectiveness of the proposed methodologies are presented.

1. Introduction

A mathematical model accurately represents the dynamics of pragmatic linear and nonlinear systems. Modeling errors and disturbances encountered due to disconcerted or lethargic parameters, data errors, environmental noises, perturbations and system age incur deviation from the original dynamics in state-estimation and control applications. Such uncertainties might lead to control-system instability, lag, mismatches or performance degradation. Consequently, robust stabilization methodologies against uncertainties in linear systems along with techniques for recouping faulty situations have been subjected to substantial experimental trials [1–7]. Still, state-feedback controller designs are crucial for dynamic real-world systems, as direct measurement of all state variables can be very, even prohibitively costly [8–10]. Thus, state-estimation-based optimal controllers have been widely adopted over the past decade for stabilization of deterministic and stochastic linear plants [7,11–13]. Also, for linear systems, observer designs and observer-based control techniques have been proposed to deal with parameter uncertainty and undesirable disturbances acting in real systems [14–18], while the counterpart nonlinear solutions are still immature. In [19], the authors studied robust H_∞ filtering for continuous-time nonlinear systems that incorporates the properties of the Lipschitz continuity under parametric uncertainties. In [20], to attain the robustness feature, the L_2-induced gain from the norm-bounded originating disturbance signals to the state-estimation errors was deduced.

For nonlinear systems, various methodologies for design of robust observers or controllers to handle exogenous disturbances have been explored [21–24]. Major examples include Kalman filtering for Gaussian measurement randomness with cognized statistics [21] as well as the H_∞ approach to arbitrary noise signals with bounded energy for ensuring the noise-evanescing level [22]. Broadly speaking, confining the difficulty of state observers to nonlinear systems, most of the studies have been carried out for particular nonlinearities. A renowned class of nonlinearity that is locally satisfied by numerous physical systems is Lipschitz nonlinearity. Several efficient observer design and state-estimation-based controller synthesis results for Lipschitzian nonlinear systems have been reported [8,25,26]. A specific state-estimation solution for a nonlinear dynamical system is provided in [25], where stability conditions, as in the algebraic Riccati equation, are explicit. In [26], an observer design for nonlinear systems and the H_∞ adaptive invariant set are presented to avoid difficulty in adaptive observer design.

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Lipschitz continuity is extended to the one-sided Lipschitz condition, a more generic one that incorporates the less specific class of nonlinearities as a special case and also abridges the conservatism in observer designs (see Refs. [27–30]). State-estimation is a less pragmatic problem than observer-based control of one-sided Lipschitz nonlinear systems. Significant work has been done on observer design and observer-based control for continuous-time and discrete-time nonlinear systems in [3,31–38]. The latest work, by contrast, encompasses observer-based stabilization of nonlinear systems in the presence of noise [24,39–42]. H_∞ filtering for singular Lipschitz nonlinear systems has been addressed in [40,41]. In [43], the sufficient condition for robust H_2 fuzzy observer-based control of T–S fuzzy models that ensures the closed-loop stability of fuzzy systems has been investigated. A less conservative linear matrix inequality (LMI)-based H_∞ observer design for one-sided Lipschitz nonlinear systems in the presence of noise is presented in [42]. However, to the best of authors' knowledge, less attention has been paid to the observer-based control of one-sided Lipschitz nonlinear systems with parametric uncertainties and disturbances.

Inspired by the aforementioned intense discussion on parametric uncertainties and one-sided Lipschitz systems, the present work explores observer-based stabilization of one-sided Lipschitz nonlinear systems with parametric uncertainties and disturbances. A Lyapunov functional is adopted, the time derivative of which further involves the one-sided Lipschitz condition and quadratic inner-boundedness condition as observer-based controller design conditions. Further, Young's relation is specifically employed to resolve the bilinear matrix inequality condition for extraction of straightforward observer-based control results and, thus, enable simultaneous computation of the controller and observer gains. On this basis, robust observer-based controller synthesis strategy under parametric uncertainties is rendered against L_2-norm-bounded exogenous disturbances to ensure L_2 gain reduction from the undesired signals to an augmented vector comprising the system's state and state-estimation error. Further, a solution to the nonlinear constraint is provided by solving the optimization problem using cone-complementary linearization approach. An observer-based robust control treatment against perturbations, disturbances and parametric uncertainties has been provided for the first time to the best of our cognizance. Finally, numerical simulation examples demonstrating the effectiveness of the proposed observer-based control scheme are presented.

This paper is organized as follows. Section 2 describes the observer-based robust control system, and Section 3 summarizes its formulation and experimental results. Sections 4 and 5 provide numerical simulation results and the concluding remarks, respectively.

Standard notation is used in this paper. $\|s\|$ denotes the Euclidean norm of a vector s; the L_2 norm of the vector is given by $\|s\|_2 = \sqrt{\int_0^\infty \|s\|^2 dt}$. For the same dimension vectors r and s, the inner product of the vectors is presented as $\langle r, s \rangle$. Moreover, the quantity $\sup_{\|d\|_2 \neq 0} (\|z\|_2 / \|d\|_2)$ defines the L_2 gain for a system with an input vector d and output vector z. To denote a symmetric positive (or semi-positive) matrix P, we use the matrix inequality $P > 0$ (or $P \geq 0$). The representation $diag(s_1, s_2, \ldots, s_n)$ denotes a block diagonal matrix with entry s_i, for $i = 1, 2, \ldots, n$, at the corresponding diagonal element. Matrix A^T represents the transpose of A, and the symbol * corresponds to a term to impart symmetry to a symmetric matrix.

2. System description

Consider the continuous-time one-sided Lipschitz uncertain nonlinear system

$$\begin{aligned} \dot{x}(t) &= (A + \Delta A(t))x(t) + f(t,x) + Bu(t) + d(t), \\ y(t) &= (C + \Delta C(t))x(t), \end{aligned} \tag{1}$$

where $x(t) \in \Re^n, y(t) \in \Re^p, d(t) \in \Re^n$ and $u(t) \in \Re^m$ represent the state, output, disturbance and control input, respectively, $f(t,x)$ represents the nonlinear dynamics associated with the state vector with $f(t,0) = 0$, and A, B and C correspond to the linear constant matrices of a system of appropriate dimensions. The unknown matrices $\Delta A(t)$ and $\Delta C(t)$ are the time-varying parametric uncertainties accounted as

$$\begin{aligned} \Delta A(t) &= M_1 F(t) N_1, \\ \Delta C(t) &= M_2 F(t) N_2, \end{aligned} \tag{2}$$

where M_1, M_2, N_1 and N_2 are known real constant matrices and $F(t)$ represents an unknown real-valued matrix function satisfying

$$F^T(t)F(t) \leq I, \quad \forall t \geq 0. \tag{3}$$

In order to apply the prelude condition of one-sided Lipschitz nonlinearity and quadratic inner-boundedness, define a functional region as $\wp \subseteq R^n$, and then establish the definitions (see [29,43] and references therein).

Definition 1. If there exists $\rho \in R \ \forall \ x_1, x_2 \in \wp$ such that

$$\langle f(t,x_1) - f(t,x_2), x_1 - x_2 \rangle \leq \rho \|x_1 - x_2\|^2, \tag{4}$$

the known nonlinear function $f(t,x)$ is said to be one-sided Lipschitz, where ρ is the one-sided Lipschitz constant.

Definition 2. If $\forall \ x_1, x_2 \in \wp$ there exist $\alpha, \beta \in R$ such that

$$\begin{aligned} &(f(t,x_1) - f(t,x_2))^T (f(t,x_1) - f(t,x_2)) \\ &\quad \leq \beta \|x_1 - x_2\|^2 + \alpha \langle x_1 - x_2, f(t,x_1) - f(t,x_2) \rangle, \end{aligned} \tag{5}$$

the nonlinear function $f(t,x)$ is said to be quadratically inner-bounded in the region $\wp$.

Every nonlinear function $f(t,x)$ is said to conform to the Lipschitz condition and then to satisfy the one-sided Lipschitz and quadratic inner-boundedness conditions, whereas the converse is not true [43]. It is worth adverting that the constants ρ, β and α can constitute zero, negative or positive values unlike the orthodox Lipschitz constant, which adopts the positive value only. Some significant examples in this regard include $-x^3$, which is a locally Lipschitz. This function is globally one-sided Lipschitz with the constant $\rho = 0$. Similarly another function $-\text{sgn}(x)\sqrt{|x|}$ is also globally one-sided Lipschitz with one-sided constant $\rho = 0$, while this function is not Lipschitz in any domain containing the origin. Note that these functions also satisfy the condition in Definition 2, while the values of α and β depend on the selection of region $\wp$.

Assumption 1. The function $f(t,x)$ fulfills the one-sided Lipschitz and quadratic inner-boundedness conditions in (4) and (5).

Assumption 2. The pairs (A, B) and (A, C) are stabilizable and detectable, respectively.

In order to conclude our results, the Lemma is introduced below.

Lemma 1. *([44]). For any constant $\nu > 0$ and known real matrices X, Γ and U of appropriate dimensions, the inequality.*

$$X\,\Gamma(t)U + [X\,\Gamma(t)U]^T \leq \nu^{-1} XX^T + \nu U^T U \tag{6}$$

holds, where $\Gamma(t)$ is a time-varying uncertain matrix fulfilling $\Gamma^T(t)\Gamma(t) \leq I$.

The observer dynamics, for the observer gain $L \in R^{n \times p}$, are in the form

$$\dot{\hat{x}}(t) = A\hat{x}(t) + f(t,\hat{x}) + Bu(t) + L(y(t) - \hat{y}(t)), \tag{7}$$

$$\hat{y}(t) = C\hat{x}(t), \tag{8}$$

where $\hat{x}(t)$ and $\hat{y}(t)$ are the estimated state and output, respectively. Defining the estimation error as $e(t) = x(t) - \hat{x}(t)$, the observer error dynamics become