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Simplified filtered Smith predictor for MIMO processes with multiple time delays

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ABSTRACT

This paper proposes a simplified tuning strategy for the multivariable filtered Smith predictor. It is shown that offset-free control can be achieved with step references and disturbances regardless of the poles of the primary controller, i.e., integral action is not explicitly required. This strategy reduces the number of design parameters and simplifies tuning procedure because the implicit integrative poles are not considered for design purposes. The simplified approach can be used to design continuous-time or discrete-time controllers. Three case studies are used to illustrate the advantages of the proposed strategy if compared with the standard approach, which is based on the explicit integrative action.

1. Introduction

The time interval between an input change and its initial effect over a related output may appear due to mass, energy and information time lags, computation time delay, and reduced order representation effect [1]. These time delays are undesired for control purposes because the current control action is not able to directly deal with the current set-point tracking error due to the dead-time interval [2]. The Smith predictor (SP) [3] was originally proposed for single-input single-output (SISO) systems in order to improve closed-loop performance. This prediction strategy provides three interesting characteristics for the nominal case: (i) dead-time compensation – time delay is removed from the characteristic equation; (ii) output prediction – for set-point changes, the predictor output represents an estimation of the future output; and (iii) ideal dynamic compensation – the plant is factorized into a product of a delay-free system and a pure time-delay part [2].

The Smith predictor was originally extended to the multiple-input multiple-output (MIMO) problems in [4]. In [5], it was shown that this extension [4] is not unique. Actually, a desired property can be ensured, depending on the MIMO dead-time compensation structure, but the original SISO properties may not hold at the same time. The dead-time compensation property, also known as delay free characteristic equation property, can be found in several works with multiple time delays [4,6–8]. Its main advantage comes from the fact that the characteristic equation is polynomial (it is not transcendental) and closed-loop stability margins are not affected by the delays. Output prediction property is also found in other important works because the predicted output vector can be analyzed as an estimation of the future output [2,5,9,10]. Recently, a generalized structure was proposed for the filtered Smith predictor in order to analyze either delay free characteristic equation or output prediction strategies in terms of internal stability. Indeed, previous related works have assumed that the process model is open-loop stable due to the original Smith predictor inherent limitation [11]. Fortunately, it has been shown that the discrete-time MIMO filtered Smith predictor (MFSP) can be also used to control open-loop unstable systems by using a suitable implementation structure [11].

As can be verified from the related works [2,4,5,7,10,11], a controller with integrative action is commonly used in MIMO Smith predictor strategies. Similar to the SISO case, this kind of choice is mainly motivated to guarantee offset-free control with step references and disturbances. Actually, as discussed in [10,11], an integral term can be explicitly used to define the control action in order to achieve these steady-state requirements. Recently, it was shown that the primary controller design can be simplified for SISO problems, avoiding the integral action, by correctly defining the static gain of two particular filters [12,13]. These results can be

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used to improve closed-loop response and to simplify design procedure [12,13].

This paper proposes an extension of the simplified tuning conditions [12,13] for MIMO processes with multiple time delays. The proposed approach can be used by considering continuous-time or discrete-time design problems in order to control open-loop stable or unstable systems. Due to the generality of the proposed strategy, either delay free characteristic equation property or output prediction property can be achieved. Simulation examples are presented to illustrate the benefits of the proposed approach.

The paper organization is presented in sequel: the MFSP background is discussed in Section 2; the proposed approach is presented in Section 3; the results of the simulation examples are shown in Section 4; and the concluding remarks are discussed in Section 5.

2. Preliminaries

In this work, a $n \times m$ multivariable transfer matrix is considered as follows:

$$\mathbf{P_n}(s) = \begin{bmatrix} G_{11}(s)e^{-sL_{11}} & G_{12}(s)e^{-sL_{12}} & \dots & G_{1m}(s)e^{-sL_{1m}} \\ G_{21}(s)e^{-sL_{21}} & G_{22}(s)e^{-sL_{22}} & \dots & G_{2m}(s)e^{-sL_{2m}} \\ \vdots & \vdots & \ddots & \vdots \\ G_{n1}(s)e^{-sL_{n1}} & G_{n2}(s)e^{-sL_{n2}} & \dots & G_{nm}(s)e^{-sL_{nm}} \end{bmatrix}, \quad (1)$$

where $G_{ij}(s)e^{-sL_{ij}}$ represents each partial transfer function relating the j-th input with the i-th output. The delay free partial transfer function is represented by $G_{ij}(s)$, and L_{ij} describes each partial delay. Also observe that these delays may appear from different sources, representing input, output, and internal coupling delays.

In order to guarantee arbitrary set-point tracking for step references, it is assumed that $n \le m$.[1] If the reference is correctly defined (reachable set-point), the assumption $n \le m$ can be relaxed, however a disturbance observer should be considered as, in practice, a reachable output which also depends on unmeasurable disturbance effects. Anyway, the case with $n > m$ is not considered in this work for presentation simplicity purposes.

Modeling error and external disturbance effects are described as follows:

$$\mathbf{Y}(s) = \mathbf{P}(s)[\mathbf{U}(s) + \mathbf{Q}_u(s)] + \mathbf{Q}_y(s), \quad (2)$$

where $\mathbf{U}(s) \in \mathcal{R}^m$ represents the input vector in s-domain, $\mathbf{Y}(s) \in \mathcal{R}^n$ describes the output vector in s-domain, $\mathbf{Q}_u(s) \in \mathcal{R}^m$ represents the unknown input disturbance and $\mathbf{Q}_y(s) \in \mathcal{R}^n$ is the unknown output disturbance vector in s-domain. Note that $\mathbf{P}(s) \ne \mathbf{P_n}(s)$ is used to describe modeling error.

The SISO Smith predictor has three important properties [2,5]: (i) output prediction, (ii) delay free characteristic equation, and (iii) ideal dynamic compensation. However, for MIMO Smith predictors, depending on the nature of the multiple delays, these properties cannot be achieved at the same time, by using the same Dead-Time Compensator (DTC) structure. As previously discussed, delay free characteristic equation holds for Smith predictor based strategies as in [4,6–8], while output prediction property is ensured in [2,5,9,10]. Recently, a unified filtered Smith predictor description was analyzed in [11], where the desired DTC property is defined by the type of the undelayed model, also known as fast model. An *output DTC fast model* is used to guarantee output compensation property and a *full DTC fast model* is considered to ensure delay free characteristic equation property.

The *output DTC fast model* is defined as follows:

$$\mathbf{G_o}(s) = \begin{bmatrix} G_{11}(s)e^{-s(L_{11}-L_1)} & G_{12}(s)e^{-s(L_{12}-L_1)} & \dots & G_{1m}(s)e^{-s(L_{1m}-L_1)} \\ G_{21}(s)e^{-s(L_{21}-L_2)} & G_{22}(s)s^{-s(L_{22}-L_2)} & \dots & G_{2m}(s)e^{-s(L_{2m}-L_2)} \\ \vdots & \vdots & \ddots & \vdots \\ G_{n1}(s)s^{-s(L_{n1}-L_n)} & G_{n2}(s)s^{-s(L_{n2}-L_n)} & \dots & G_{nm}(s)e^{-s(L_{nm}-L_n)} \end{bmatrix}, \quad (3)$$

where $L_i = \min_{j=1,\dots,m} L_{ij}$, for $i = 1,\dots,n$. Note that, in this case, a multivariable effective output delay can be defined by:

$$\mathbf{L}(s) = \begin{bmatrix} e^{-sL_1} & 0 & \dots & 0 \\ 0 & e^{-sL_2} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & e^{-sL_n} \end{bmatrix}.$$

Hence, the process model can be decomposed as follows:

$$\mathbf{P_n}(s) = \mathbf{L}(s)\mathbf{G_o}(s) \to \mathbf{G_o}(s) = \mathbf{L}(s)^{-1}\mathbf{P_n}(s). \quad (4)$$

Thus, for the nominal case, i.e., without modeling error ($\mathbf{P_n}(s) = \mathbf{P}(s)$) and without disturbances, Eq. (2) can be expressed by:

$$\mathbf{Y}(s) = \mathbf{P_n}(s)\mathbf{U}(s) = \mathbf{L}(s)\mathbf{G_o}(s)\mathbf{U}(s). \quad (5)$$

Therefore, a nominal prediction for $\mathbf{Y}(s)$, namely $\mathbf{Y_p}(s) \in \mathcal{R}^n$, can be obtained as follows:

$$\mathbf{Y_p}(s) = \mathbf{G_o}(s)\mathbf{U}(s) = \mathbf{L}(s)^{-1}\mathbf{P_n}(s)\mathbf{U}(s) = \mathbf{L}(s)^{-1}\mathbf{Y}(s). \quad (6)$$

Observe that the relationship between $\mathbf{Y_p}(s)$ and $\mathbf{U}(s)$ has no (effective) output delay. As previously pointed out, this type of fast model can be found in [2,5,9,10].

In order to achieve delay-free characteristic equation property, the following *full DTC fast model* is used:

$$\mathbf{G_f}(s) = \begin{bmatrix} G_{11}(s) & G_{12}(s) & \dots & G_{1m}(s) \\ G_{21}(s) & G_{22}(s) & \dots & G_{2m}(s) \\ \vdots & \vdots & \ddots & \vdots \\ G_{n1}(s) & G_{n2}(s) & \dots & G_{nm}(s) \end{bmatrix}. \quad (7)$$

In this case, the nominal characteristic equation is polynomial (without delays), but $\mathbf{Y_p}(s) = \mathbf{G_f}(s)\mathbf{U}(s)$ is not necessarily a prediction for $\mathbf{Y}(s)$. This kind of fast model was already used in other works [4,6–8].

2.1. Multivariable filtered Smith predictor

The unified filtered Smith predictor description was proposed to analyze *output DTC* and *full DTC* strategies with respect to internal stability. The analysis structure can be presented either in continuous or discrete-time. The continuous-time version of this generalized description is depicted in Fig. 1(a) where $\mathbf{G_\bullet}(s)$ describes the fast model, $\mathbf{F_r}(s)$ is a $n \times n$ robustness filter, $\mathbf{F}(s)$ is the $n \times n$ reference filter, and $\mathbf{C}(s)$ is the $m \times n$ primary controller. The notation $\mathbf{G_\bullet}(s)$ is used in order that $\mathbf{G_\bullet}(s) = \mathbf{G_o}(s)$ describes the *output DTC fast model* and $\mathbf{G_\bullet}(s) = \mathbf{G_f}(s)$ is used to analyze the *full DTC fast model* approach.

It was already shown that an internally stable discrete-time implementation structure should be used if the process is not open-loop stable [11]. The internally stable implementation structure in depicted in Fig. 1(b) where $\mathbf{S_\bullet}(z) = [\mathbf{G_\bullet}(z) - \mathbf{F_r}(z)\mathbf{P_n}(z)]$ with $\mathbf{G_\bullet}(z)$ and $\mathbf{P_n}(z)$ obtained from the zero-order hold (ZoH) discretization of $\mathbf{G_\bullet}(s)$ and $\mathbf{P_n}(s)$, respectively. Actually, the BIBO[2] stability of $\mathbf{S_\bullet}(z)$ is a necessary condition for internal stability [11]. For the case of open-loop stable models, $\mathbf{S_\bullet}(s)$ or $\mathbf{S_\bullet}(z)$ are BIBO stable transfer matrices if $\mathbf{F_r}(s)$ or $\mathbf{F_r}(z)$ are also BIBO stable transfer matrices. If the process model is not open-loop stable, $\mathbf{F_r}(z)$ may be

[1] Note that if the number of outputs is greater than the number of inputs, a given reference vector may not be reachable.

[2] BIBO stands for Bounded-Input Bounded-Output.

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