ISA Transactions ∎ (∎∎∎∎) ∎∎∎–∎∎∎

Contents lists available at ScienceDirect

ISA Transactions

journal homepage: www.elsevier.com/locate/isatrans

# Hybrid controller with observer for the estimation and rejection of disturbances

José de Jesús Rubio

*Sección de Estudios de Posgrado e Investigación, ESIME Azcapotzalco, Instituto Politécnico Nacional, Av. de las Granjas no. 682, Col. Santa Catarina, México D.F. 02250, Mexico*

A R T I C L E I N F O

*Article history:*
Received 7 May 2016
Received in revised form
12 July 2016
Accepted 3 August 2016
This paper was recommended for publication by Dr. Jeff Pieper

*Keywords:*
Sliding mode
Output feedback
Controller
Observer
Stability
Disturbance
Plotter
Suspension system

A B S T R A C T

In this paper, a hybrid controller with observer is introduced for the estimation and rejection of a disturbance. It is based on the combination of the sliding mode technique and the output feedback strategy. It is divided into two designs: (1) the observer and (2) the controller with observer. The observer is selected to reach two objectives: (a) to assure its stability and (b) for the estimation of a disturbance. The controller with observer is selected to reach three objectives: (a) to assure its stability, (b) for the rejection of a disturbance, and (c) for the decreasing of chattering in the sliding mode behavior. The proposed method is applied for the estimation and rejection of the disturbance in a plotter and a suspension system.



## 1. Introduction

Disturbances are undesired signals presented as inputs of a system which affect outputs. This problem has been studied in many systems. Rejection of a disturbance in the output of a system is required because it can affect the sensors, actuators, plants, or controls, causing accidents or unnecessary costs. Thus, a control to reject or to attenuate the disturbance in the output of a system is an important and current issue, in the theory and applications.

There is some research about the disturbance rejection or about controllers with observers. The criteria of robustness for a controller to disturbances are evaluated in [1]. In [2], the optimal tracking performance of disturbed plants is considered. Various methods of improving the overall estimation quality in state observers are considered in [3]. In [4,5], the sliding mode approach is utilized for the disturbance rejection. Methods for the disturbance detection are presented in [6,7]. In [8], a novel control scheme based on disturbance rejection is mentioned. A observer based controller using the output feedback is designed in [9]. In [10], an adaptive fuzzy backstepping output-feedback tracking control approach is proposed. An adaptive fuzzy robust output feedback control problem is considered in [11]. In [12], a hybrid fuzzy adaptive output feedback control design approach is introduced. A composite adaptive fuzzy output-feedback control approach is suggested in [13]. In [14], a fuzzy adaptive output-feedback stabilization control method is designed. A control for the stabilization is introduced in [15]. In [16], a fault-tolerant control of fuzzy systems is mentioned.

The above research, mention some important investigations in controllers for the disturbance rejection such as [1–8], or in controllers with observers for the trajectory tracking such as [10–15]. However, it is not common to design controllers with observers for the estimation and rejection of a disturbance such as [9,16].

In this study, a controller with observer is suggested for the disturbance rejection. The introduced strategy is divided into two designs, the observer and the controller with observer. The observer is selected to reach two main objectives: (1) to assure its stability by the utilization of the Lyapunov technique and (2) for the behavioral estimation of the unknown disturbance. The controller with observer is selected to reach three main objectives: (1) to assure its stability by the utilization of the Lyapunov technique, (2) for the rejection or attenuation of the unknown disturbance, and (3) to suppose an alternative for the decreasing of chattering in the sliding mode behavior.

*E-mail addresses:* jrubioa@ipn.mx, rubio.josedejesus@gmail.com

http://dx.doi.org/10.1016/j.isatra.2016.08.026
0019-0578/© 2016 ISA. Published by Elsevier Ltd. All rights reserved.

On the other hand, the aforementioned research shows that the disturbance rejection problem is an actual and novel investigation. In this study, an alternative method for the estimation and rejection of disturbance is designed, the method is based on the combination of the sliding mode technique [17–21] and the output feedback strategy [22–26].

The sliding mode controller has two problems: (1) the produced chattering because it can damage devices and (2) the sliding mode surface because it requires many derivatives of the system. A proposal of solution for both problems is presented in this research as follows: (1) the chattering of the sliding mode controller is decreased by utilizing the estimated disturbance from an observer and (2) derivatives of the system are not required because the sliding mode surface of this controller is changed by the output feedback strategy.

And finally, there are some works about stable disturbance rejection. In [27], the disturbance rejection of closed loop control systems is focused. Robust controls of turbines with disturbances are introduced in [28–30]. In [31], the stabilization for a Schrodinger equation with a disturbance is designed. The stability using a disturbance rejection control is approached in [32]. In [33], a robust controller of disturbed systems is designed. The sliding mode observer for jump systems against actuator faults is investigated in [34]. In [35], the disturbance rejection control for stable systems is investigated. The H-infinity control of a neural network with an external disturbance is designed in [36]. In [37], the robust output feedback control of systems with disturbances is explained. The robust stability of a class of disturbed systems is investigated in [38].

From the above studies, in most of the cases authors use the Lyapunov method. Therefore, in this investigation, the Lyapunov analysis is utilized to assure the exponential stability of the proposed controller with observer.

This paper is organized as follows. Section 2 presents the employed systems, they will be used for the controller design. In Section 3, the hybrid observer is proposed for the disturbance estimation. In Section 4, the hybrid controller with observer is proposed for the disturbance rejection. In Sections 5 and 6, the introduced strategy is verified by two examples: the plotter and suspension systems, respectively. Section 7 presents conclusions and suggests future research directions.

## 2. The disturbed system

Consider the following disturbed system:

$$\dot{z} = Az + Bv + Bd$$
$$y = Cz \tag{1}$$

where $z \in \Re^n$ is the state, $v \in \Re^m$ is the control input, $y \in \Re^p$ is the output, $d \in \Re^m$ is the disturbance. $A \in \Re^{n\times n}$, $B \in \Re^{n\times m}$, and $C \in \Re^{p\times n}$ are matrices representing the transformations $\mathbf{A}: \Re^n \to \Re^n$, $\mathbf{B}: \Re^m \to \Re^n$, and $\mathbf{C}: \Re^n \to \Re^p$, respectively.

The following two sections will describe the observer for the estimation of a disturbance, and the controller with observer for the rejection or attenuation of a disturbance.

## 3. Hybrid observer

### 3.1. Observer design

In this subsection, a hybrid observer will be designed based on the assumption that the output $y$ is known, and the disturbance $d$ is unknown. Let $\hat{z} \in \Re^n$ be the estimation of states $z \in \Re^n$. Define the output error $\tilde{y}$ as follows:

$$\tilde{y} = y - \hat{y} = C\tilde{z} \tag{2}$$

where $y = Cz \in \Re^p$, $\hat{y} = C\hat{z} \in \Re^p$, $\tilde{z} = z - \hat{z} \in \Re^n$ is the state error.

The following hybrid observer is proposed:

$$\dot{\hat{z}} = A\hat{z} + K\tilde{y} + Bv + N\,\mathrm{sgn}(M\tilde{y}) \tag{3}$$

where $\tilde{y}$ is the output error, $N \in \Re$ is a constant matrix which is selected after, $M$ is a vector or matrix such that $MC \in \Re^{n\times n}$ is a positive semi-definite constant, $K$ is a vector or matrix such that $KC \in \Re^{n\times n}$ is a constant matrix, sgn( · ) is the signum function, $M$, $N$, and $K$ will be utilized in the stability analysis of the next subsection.

**Remark 1.** The observer of Eqs. (2) and (3) is selected to find two main objectives: (1) to assure its stability by the utilization of the Lyapunov technique in the application of systems with the form (1) and (2) for the behavioral estimation of the unknown disturbance in the system (1).

The objective of the observer is that states $\hat{z}$ and the disturbance $\hat{d}$ of (3) may follow states $z$ and the disturbance $d$ of (1) using outputs $y$ in the output error (2). See Fig. 1.

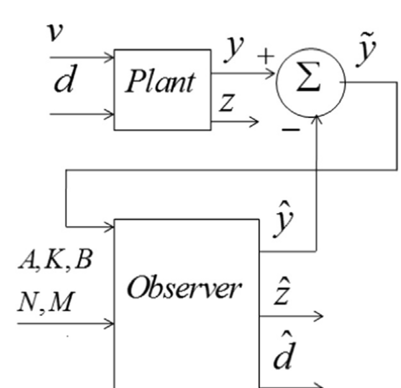

**Fig. 1.** Hybrid observer.

### 3.2. Stability analysis

In this subsection, the exponential stability of the closed loop in the hybrid observer and the disturbed system is assured by the solution of the Lyapunov method.

From (1) to (3), the observer error equation can be formed as follows:

$$\begin{aligned}\dot{\tilde{z}} &= A\tilde{z} + Bd - K\tilde{y} - N\,\mathrm{sgn}(M\tilde{y})\\ &= [A - KC]\tilde{z} + Bd - N\,\mathrm{sgn}(M\tilde{y})\\ &= A_o\tilde{z} + Bd - N\,\mathrm{sgn}(M\tilde{y})\end{aligned} \tag{4}$$

where $A_o = A - KC$.

The following theorem shows the stability of the proposed hybrid observer.

**Theorem 1.** *The state of the hybrid observer (2), (3) is applied to estimate the state $z$ of the disturbed system (1) and it ensures exponential stability; thus, the observer error $\tilde{z}$ satisfies:*

$$\|\tilde{z}\|^2 \le \alpha e^{-\gamma t}\|\tilde{z}_i\|^2 \tag{5}$$

*where $\tilde{z}_i$ is the initial condition of $\tilde{z}$, $\alpha = \frac{\lambda_{\max}(S)}{\lambda_{\min}(S)}$, $\gamma = \lambda_{\min}(RS^{-1})$, $\|d\| \le \bar{d}$, $\bar{d}$ is the known upper bound of $d$, $d$ is the unknown disturbance given in (1), $\|Bd\| \le \|B\|\bar{d} \le N$, $MC \ge 0$, $\|\cdot\|$ is the Euclidean norm in $\Re^n$ and $|\cdot|$ is the absolute value, $S \in \Re^{n\times n}$ and $R \in \Re^{n\times n}$ are*

Please cite this article as: Rubio JdJ. Hybrid controller with observer for the estimation and rejection of disturbances. ISA Transactions (2016), http://dx.doi.org/10.1016/j.isatra.2016.08.026