ISA Transactions ∎ (∎∎∎∎) ∎∎∎–∎∎∎

Contents lists available at ScienceDirect

ISA Transactions

journal homepage: www.elsevier.com/locate/isatrans

State-dependent switching control of switched positive fractional-order systems☆

Xudong Zhao [a,b,*], Yunfei Yin [a], Xiaolong Zheng [a]

[a] College of Engineering, Bohai University, Jinzhou, Liaoning 121013, China
[b] Chongqing SANY High-Intelligent Robots Co., Ltd., Chongqing 401120, China

ARTICLE INFO

Article history:
Received 16 June 2015
Received in revised form
30 November 2015
Accepted 15 January 2016
This paper was recommended for publication by Mohammad Haeri

Keywords:
Fractional-order systems
Positive system
State-dependent switching
Sliding sector
Variable structure

ABSTRACT

In this paper, the problem of switching stabilization for a class of continuous-time switched positive fractional-order systems is studied by using state-dependent switching. First, the asymptotic stability condition of switched positive fractional-order systems with state-dependent switching is given, which is based on the fractional co-positive Lyapunov method. Moreover, by the sliding sector method, the stability condition of switched positive fractional-order systems whose subsystems are possibly all unstable is obtained. A variable structure (VS) switching law with sliding sector is also proposed to guarantee the switched positive fractional-order system to be asymptotically stable. Finally, two numerical examples are given to demonstrate the advantages and effectiveness of our developed results.

1. Introduction

Many physical systems encountered in practice involve state variables that always are confined to be nonnegative. For instance, the density of the object, absolute temperatures, concentration of substances in chemical processes and the number of population in biology are always positive. Such systems are generally termed as positive systems whose states and output are positive (at least nonnegative) whenever the initial conditions and input are nonnegative [1–5]. Recently, the importance of switched positive systems [6] has also been highlighted by a lot of researchers mainly because of their wide applications such as biology, formation flying, networks employing TCP and communication networks. Switched positive systems are composed of continuous-time or discrete-time positive subsystems and discrete switching events [7]. In recent years, a larger number of well-developed results of switched positive systems have been presented in a mass of works. As far as the stability of switched positive systems is concerned, the authors of [8] have shown that a common linear co-positive Lyapunov function (CLCLF) can guarantee the system asymptotic stability under arbitrary switching. In [4], the authors proposed a kind of co-positive polynomial Lyapunov function (CPLF) to investigate the problems of stability for switched positive linear systems under arbitrary switching. However, these still may, to some extent, lead to some conservativeness since quite a number of practical systems fail to preserve stability under arbitrary switching. Then, the authors in [5] applied a class of multiple linear co-positive Lyapunov function (MLCLF) to study the problem of switching stabilization for such systems.

Meanwhile, considerable attention has been paid to fractional-order systems due to its many applications in almost all applied sciences such as dynamical processes in self-similar structures, chemical engineering, control of power electronics, dynamics of earthquakes and signal processing [9–13]. Fractional-order calculus naturally has hereditary properties and long memory transients, and is also an extension and promotion of integer-order calculus concept, which can better describe practical switched positive systems. Therefore, studies on switched positive fractional-order systems causally have become a very active area. In particular, stability analysis of switched positive fractional-order systems has attracted relatively more attention [14–17]. However, it should be pointed out that different from switched positive integer-order systems, only sufficient conditions could be obtained on analysis of stability problems for switched positive fractional-order systems since it is composed of a set of positive fractional-order subsystems [15]. That is to say, those methods, such as the

☆This work was partially supported by the National Natural Science Foundation of China (61203123, 61573069) the Liaoning Excellent Talents in University (LR2014035), and the Liaoning Provincial Natural Science Foundation, China (2015020053).

* Corresponding author.
E-mail addresses: xdzhaohit@gmail.com (X. Zhao), yyfeillei@gmail.com (Y. Yin), zxldwlsj@gmail.com (X. Zheng).

http://dx.doi.org/10.1016/j.isatra.2016.01.011
0019-0578/

popular common linear co-positive Lyapunov function used in [8] and the multiple linear co-positive Lyapunov function adopted in [5] for switched positive integer-order systems, still leave rooms for improvements. Although considering the nature of positivity, these methods fail to capture the intrinsic properties of a fractional-order system. Thus, it will be challenging and interesting to study switched positive fractional-order systems.

On the other hand, when a switched positive fractional-order system is composed of subsystems that may be all unstable, aforementioned results clearly are not feasible, because there is not stable subsystems to compensate the state divergence caused by unstable subsystems. Hence, an efficient switching signal needs to be designed to stabilize the switched positive fractional-order systems with unstable subsystems. Variable structure (VS) switching signal that can be viewed as one of constraint switching signals, naturally has been extensively investigated during the past several decades [18,19]. It is generally known that the so-called variable structure control is a form of discontinuous nonlinear control, and it changes the dynamics of a system by applying a switching control. Sliding motion occurs when the system state repeatedly crosses a certain subspace in the state space, which can be seen as one of the most important features of VS control. A sliding sector has been proposed to replace the sliding mode in discrete-time control systems in [20]. Furthermore, the authors of [19] presented a VS controller with sliding sector for switched systems. It has been proven in [21] that for any system there is always a sliding sector in the state space which can be designed by using the Riccati equation. However, to the best of our knowledge, up to now, there has been not any literature studying the problem of stabilization of switched positive fractional-order systems composed of unstable subsystems via VS switching.

All the above observations motivate us to carry out the present study: based on the common linear co-positive Lyapunov function theory, propose a new VS control law that is different from the traditional VS control law to solve the switching stabilization for a given switched positive fractional-order system composed of possibly all unstable modes to improve some existing results on this issue.

In this paper, the problems of stability and stabilization for switched positive fractional-order systems with a new class of switching signals will be studied in continuous-time context. A novel state dependent switching signal is proposed for switching stabilization design of switched positive fractional-order systems. In addition, the stability and stabilization conditions of the systems with new VS control law are derived by using fractional common co-positive Lyapunov functions. The remainder of the paper is organized as follows. Section 2 reviews some necessary definitions and lemmas of fractional-order system and positive system. In Section 3, stabilization criteria for switched positive fractional-order systems with state dependent switching are derived, upon which some improved conditions of the considered systems are also developed by using novel VS control law. Section 4 provides two numerical examples to demonstrate the feasibility and effectiveness of our proposed techniques, and Section 5 concludes the paper.

Notations: In this paper, the notations used are standard. $A \succcurlyeq 0$ (or $A \succ 0$) denotes that all entries of matrix A are positive (or nonnegative); $x \succcurlyeq 0$ (or $x \succ 0$) means that all entries of vector x are positive (or nonnegative); and *Metzler* is a matrix whose off diagonal entries are nonnegative. $\mathbb{R}$, $\mathbb{R}^n$ and $\mathbb{R}^n_+$ denote the field of real numbers, n-dimensional Euclidean space and the nonnegative orthant of $\mathbb{R}^n$ respectively; the notation $\|\cdot\|$ refers to the Euclidean norm. A function α: $[0,\infty) \to [0,\infty)$ is said to be of class $\mathcal{K}$ if it is continuous, strictly increasing, and $\alpha(0)=0$. Class $\mathcal{K}_\infty$ denotes the subset of $\mathcal{K}$ consisting of all those functions that are unbounded. In addition, A^T stands for the transpose of matrix A; the notation $P>0$ (≥ 0) means that P is a real symmetric and positive definite (semi-positive definite) matrix.

2. Preliminaries

This section presents some definitions and preliminary results which will be used later. Without loss of generality, it is assumed that the lower limit of the fractional integrals and derivatives is 0 throughout this paper. There are different definitions of the fractional order integral or derivative. The Riemann–Liouville fractional derivative of $f(t)$ is defined by

$$_{R}D^{\alpha}f(t)=\frac{1}{\Gamma(1-\alpha)}\frac{d}{dt}\left(\int_0^t\frac{f(\tau)}{(t-\tau)^{\alpha}}\,d\tau\right),\quad 0<\alpha\leq 1, \tag{1}$$

where $\alpha\in\mathbb{R}$ is the order of the fractional derivative and the Gamma function generalizing factorial for non-integer arguments is defined by

$$\Gamma(\alpha)=\int_0^{\infty}e^{-t}t^{\alpha-1}\,dt. \tag{2}$$

Furthermore, define the Caputo fractional derivative as follows:

$$_{C}D^{\alpha}f(t)=\frac{1}{\Gamma(1-\alpha)}\int_0^t\frac{f'(\tau)}{(t-\tau)^{\alpha}}\,d\tau,\quad 0<\alpha\leq 1, \tag{3}$$

where f' is the first order derivative of function f.

Remark 1. From the above two definitions, we can obtain that the relation between the Caputo and Riemann–Liouville fractional derivatives is

$$_{R}D^{\alpha}f(t)={}_{C}D^{\alpha}f(t)+\frac{t^{-\alpha}}{\Gamma(1-\alpha)}f(0). \tag{4}$$

As is clear from the above descriptions, one can immediately get $_{C}D^{\alpha}f(t)\leq {}_{R}D^{\alpha}f(t)$, if $f(0)>0$.

Consider the following switched fractional-order system:

$$\begin{cases} D^{\alpha}x(t)=A_{\sigma(t)}x(t)\\ x(t_0)=x_0\\ 0<\alpha\leq 1\end{cases} \tag{5}$$

where $x(t)\in\mathbb{R}^n$ is the state vector, $D^{\alpha}x(t)$ represents the Riemann–Liouville or Caputo fractional derivative of $x(t)$ and α is the fractional order; $\sigma(t)$ is a switching signal which is a piecewise constant function from the right of time and takes its values in the finite set $S=\{1,\dots,m\}$, where $m>1$ is the number of subsystems [22]. The matrix $A_p\in\mathbb{R}^{n\times n}$, $\forall\sigma(t)=p\in S$ with appropriate dimensions represents the pth subsystem or pth mode of (5).

Definition 1 (*Kaczorek [23]*). Given any nonnegative initial condition $x_0\in\mathbb{R}^n_+$, system (5) is said to be positive if the corresponding trajectory $x(t)\in\mathbb{R}^n_+$ for all $t\geq 0$.

Lemma 1 (*Benzaouia et al. [24]*). *Continuous-time fractional-order system (5) is positive if and only if matrix A is a Metzler matrix.*

Lemma 2 (*Li et al. [25]*). *Let $x=0$ be an equilibrium point for the system $D^{\alpha}x(t)=Ax(t)$. Assume that there exists a Lyapunov function $V(t,x(t))$ and class-$\mathcal{K}$ functions β_i, $i=1,2,3$, satisfying*

$$\begin{cases}\beta_1(\|x(t)\|)\leq V(t,x)\leq\beta_2(\|x(t)\|)\\ _{C}D^{\alpha}V(x(t))\leq-\beta_3(\|x(t)\|)\end{cases} \tag{6}$$