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Iterated non-linear model predictive control based on tubes and contractive constraints

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ABSTRACT

This paper presents a predictive control algorithm for non-linear systems based on successive linearizations of the non-linear dynamic around a given trajectory. A linear time varying model is obtained and the non-convex constrained optimization problem is transformed into a sequence of locally convex ones. The robustness of the proposed algorithm is addressed adding a convex contractive constraint. To account for linearization errors and to obtain more accurate results an inner iteration loop is added to the algorithm. A simple methodology to obtain an outer bounding-tube for state trajectories is also presented. The convergence of the iterative process and the stability of the closed-loop system are analyzed. The simulation results show the effectiveness of the proposed algorithm in controlling a quadcopter type unmanned aerial vehicle.

1. Introduction

Model predictive control (MPC) refers to a class of algorithms in which models of the plant are used to predict the future behavior of the system over a prediction horizon. It is formulated by solving an on-line optimization problem. The optimal control input sequence is calculated by minimizing an objective function subject to constraints. Only the first element of the computed optimal control input is applied to the plant according to a receding horizon strategy [1,2]. Linear MPC has been successfully applied in a variety of cases due to its ability to explicitly incorporate the system model and state/inputs constraints into the control calculation [3–6].

In the last few decades, MPC principles have been extended to non-linear processes yielding to non-linear model predictive control (NMPC). The use of general non-linear programming (NLP) techniques to solve the NMPC problem has been proposed in several works [7–10]. However, the solution methods based on NLP present some drawback. First, these algorithms are computationally demanding, as they require to solve on-line a non-linear optimization problem. Moreover, the constraints introduced by the non-linear model dynamics yield to non-convex optimization problems.

Linearization and linear approximation have been adopted in a variety of works to overcome the computational complexity problem [11,12]. The main advantage of these methods lies in the fact that the model used in the prediction calculation is a set of local linear approximation of the dynamics of the plant, thus converting the non-linear optimization problem into a set of locally convex ones, as it is done in [13–15]. However, linear predictive control techniques do not automatically ensure the stability of the closed-loop system. This issue has been studied by numerous researchers for many years (see [11,16] for an overview). One way to address the stability problem is to add a contractive constraint to the optimization problem. This idea was firstly introduced by Yang and Polak [17] and the stability proof was developed by De Olivera and Morari [18]. In this approach, the authors propose to add a contractive constraint that forces the system states to decrease at each time step. To the best of our knowledge, there are few works that address the addition of such contractive constraint and also this constraint has only been used to contract the system states.

In this paper we present a novel robust predictive control algorithm for non-linear systems. The proposed algorithm uses a linearization process along pre-defined trajectories that transform the non-convex optimization problem into a set of locally convex ones, which can be solved using the standard quadratic programming (QP) techniques. Here, to address stability and convergence issues, the addition of a set of contractive constraints to the optimization problem is

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analyzed. These constraints force the cost functions to decrease or (at least) to remain constant within the current time instant, thus allowing us to take into account disturbances and determining an upper bound of the cost functions value. Moreover, an inner iteration loop is added to the proposed algorithm to account for linearization errors and to obtain more accurate results.

The organization of this paper is as follows: in Section 2 the formulation of the NMPC algorithm with the addition of the contractive constraint is presented. In Section 3 a simple methodology to obtain an outer bounding-tube for state trajectories is analyzed. In Section 4 an inner iteration loop is added to the previous algorithm. Simulation results are shown in Section 5. Finally, conclusions are discussed in Section 6.

2. Non-linear model predictive control formulation

Consider the discrete non-linear system

$$x_{k+1} = f(x_k, u_k, d_k) \tag{1}$$

where $x_k = x(k) \in \Re^n$, $u_k = u(k) \in \mathcal{U} \subseteq \Re^m$ and $d_k = d(k) \in \mathcal{D} \subseteq \Re^l$ are the state vector, the control input vector and the bounded disturbance vector, respectively, $\mathcal{U}$ is the input constraint set and $f(\cdot)$ is a continuous and differentiable vector function that describes the dynamics of the system.

The non-linear model predictive control problem is formulated as a regulatory problem stated as follows:

For a given[1] disturbance sequence

$$\mathbf{d}_k = \left[d_{k|k}, \ldots, d_{k+N-1|k}\right]^T, \tag{2}$$

find at each time instant k, a control input sequence

$$\mathbf{u}_k = \left[u_{k|k}, \ldots, u_{k+N-1|k}\right]^T, \tag{3}$$

and predicted state sequence

$$\mathbf{x}_k = \left[x_{k+1|k}, \ldots, x_{k+N|k}\right]^T, \tag{4}$$

over a prediction horizon of N sampling intervals, such that

$$\min_{\mathbf{u}_k \in \mathcal{U}} \mathcal{J}(k) \quad \text{s.t.} \quad x_{k+1} = f(x_k, u_k, d_k). \tag{5}$$

The vectors $d_{k+i|k}$, $u_{k+i|k}$ and $x_{k+i|k}$ in Eqs. (2), (3) and (4) represent the disturbance, input and state vectors respectively at time $k+i$ that are predicted using the information available at time k.[2] The optimal solution of the problem (5) is denoted here as

$$\mathbf{u}_k^* = \left[u^*_{k|k}, \ldots, u^*_{k+N-1|k}\right]^T. \tag{6}$$

Regardless of the cost function $\mathcal{J}(k)$ is convex or not, the optimization problem (5) is non-convex due to the non-linearity of the system dynamics, and the computational effort is a major issue in its on-line implementation. If $\mathcal{J}(k)$ is chosen to be a quadratic cost function, then the convexity of (5) can be recovered by approximating the non-linear model (1) with a linear time-varying (LTV) one [19,20], which can be obtained linearizing the system around a desired state and input trajectory $\mathbf{x}_k^r$, $\mathbf{u}_k^r$, where

$$\mathbf{x}_k^r = \left[x^r_{k+1|k}, \ldots, x^r_{k+N|k}\right]^T, \tag{7}$$

and

$$\mathbf{u}_k^r = \left[u^r_{k|k}, \ldots, u^r_{k+N-1|k}\right]^T. \tag{8}$$

[1] If $\mathbf{d}_k$ is not available, the most common assumption is $\mathbf{d}_{k+i} = \mathbf{d}_{k+i-1}, i = 1, \ldots, N$.

[2] When it clearly refers to current time k, the time dependency at which the information is available will be omitted, i.e. $(\cdot)_{k+i|k} = (\cdot)_{k+i}$.

Assuming that a reference perturbation $d^r_{k+i|k}$, $i = 0, \ldots, N-1$ is given or estimated, then the dynamic behavior of the deviation from the desired trajectory can be written as an LTV model

$$\tilde{x}_{k+1|k} = A_{k|k}\tilde{x}_{k|k} + B_{u_{k|k}}\tilde{u}_{k|k} + B_{d_{k|k}}\tilde{d}_{k|k}, \tag{9}$$

where

$$\tilde{x}_{k|k} = x_{k|k} - x^r_{k|k}, \quad \tilde{u}_{k|k} = u_{k|k} - u^r_{k|k} \quad \text{and} \quad \tilde{d}_{k|k} = d_{k|k} - d^r_{k|k}. \tag{10}$$

The matrices $A_{k|k}$, $B_{u_{k|k}}$ and $B_{d_{k|k}}$, are the Jacobian matrices of the discrete non-linear system (1), and they are defined as follows:

$$A_{k|k} = \left.\frac{\partial f(x_k, u_k, d_k)}{\partial x_k}\right|_{(*)}, \quad B_{u_{k|k}} = \left.\frac{\partial f(x_k, u_k, d_k)}{\partial u(k)}\right|_{(*)},$$
$$B_{d_{k|k}} = \left.\frac{\partial f(x_k, u_k, d_k)}{\partial d(k)}\right|_{(*)}, \tag{11}$$

where $(*)$ stands for (x_k^r, u_k^r, d_k^r). In terms of the LTV system (9), the following quadratic objective function $\mathcal{J}(k)$, commonly used in the literature, is adopted

$$\mathcal{J}(k) = \sum_{i=0}^{N-1}\left[\tilde{x}^T_{k+i|k}Q\tilde{x}_{k+i|k} + \tilde{u}^T_{k+i|k}R\tilde{u}_{k+i|k}\right] + \tilde{x}^T_{k+N|k}P_{k|k}\tilde{x}_{k+N|k}, \tag{12}$$

where $Q, R, P_{k|k}$ are positive definite matrices; $P_{k|k}$ is the terminal weight matrix that is chosen so as it satisfies the Lyapunov equation

$$P_{k|k} - A^T_{k|k}P_{k|k}A_{k|k} = Q. \tag{13}$$

As a result, the non-convex optimization problem (5) can be rewritten as a convex optimization problem as follows:

$$\min_{\tilde{\mathbf{u}}_k \in \mathcal{U}} \mathcal{J}(k)$$

$$\text{s.t.} \quad \begin{cases} \tilde{x}_{k+1|k} = A_{k|k}\tilde{x}_{k|k} + B_{uk|k}\tilde{u}_{k|k} + B_{dk|k}\tilde{d}_{k|k}, \\ \tilde{x}_{k|k} = x_{k|k} - x^r_{k|k}, \\ \tilde{u}_{k|k} = u_{k|k} - u^r_{k|k}, \\ \tilde{d}_{k|k} = d_{k|k} - d^r_{k|k}. \end{cases} \tag{14}$$

In Algorithm 1 the NMPC receding horizon control technique is summarized.

Algorithm 1. NMPC algorithm.

Given $Q, R > 0$, $x_{k|k}$ the initial condition.

Step 1: Obtain the linearization trajectory $\mathbf{x}_k^r$, $\mathbf{u}_k^r$ using as initial condition $\mathbf{u}_k^0 = [u^*_{k|k-1}, u^*_{k+1|k-1}, \ldots, u^*_{k+N-2|k-1}, 0]^T$ and estimate d_{k+i} for $i = 0, \ldots, N-1$

Step 2: Obtain the LTV system (9) and $P_{k|k}$ solving (13)

Step 3: Compute the optimal control input sequence $\tilde{\mathbf{u}}_k^*$ solving (14)

Step 4: Update $\mathbf{u}_k^* \leftarrow \mathbf{u}_k^r + \tilde{\mathbf{u}}_k^*$

Step 5: Apply $u_{k|k} = u^*_{k|k}$ to the system

Step 6: Move the horizon forward to the next sampling instant $k \leftarrow k+1$ and go back to **Step 1**

Linearization techniques are the most straightforward ways to adapt linear control methods to non-linear control problems. In the absence of perturbations and linearization errors, Algorithm 1 will guarantee the closed-loop stability.

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