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Research Article

Perturbed dynamics of discrete-time switched nonlinear systems with delays and uncertainties

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ABSTRACT

This paper addresses the dynamics of a class of discrete-time switched nonlinear systems with time-varying delays and uncertainties and subject to perturbations. It is assumed that the nominal switched nonlinear system is robustly uniformly exponentially stable. It is revealed that there exists a maximal Lipschitz constant, if perturbation satisfies a Lipschitz condition with any Lipschitz constant less than the maximum, then the perturbed system can preserve the stability property of the nominal system. In situations where the perturbations are known, it is proved that there exists an upper bound of coefficient such that the perturbed system remains exponentially stable provided that the perturbation is scaled by any coefficient bounded by the upper bound. A numerical example is provided to illustrate the proposed theoretical results.

1. Introduction

As an important class of hybrid dynamic systems, switched systems inherit the feature of both continuous state and discrete state dynamic systems. Roughly speaking, a switched system consists of a family of dynamical subsystems and a rule, called a switching signal, that determines the switching manner among the subsystems [1,2]. Many dynamic systems can be modeled as switched systems [3,4] which possess rich dynamics due to the multiple subsystems and various possible switching signals [5–8].

In the present paper, we are interested in the dynamics of a class of discrete-time switched nonlinear time-varying systems with delays and uncertainties. Our motivations come from the following aspects. Firstly, parameters in many practical systems vary from time to time, such as resistance and inductance in a circuit [9]. Therefore, time-varying systems have been one of the focuses of control theory for a long time [10,11]. Secondly, it is observed that any system in the real world is with certain uncertainties [12] and that delays, especially time-varying ones, inevitably appear in systems' mathematic models. It is also observed that both uncertainties and delays may lead to performance deterioration and system's malfunction [13–15]. Hence, there is a vast literature investigating various stabilities of switched systems with uncertainties and delays [16,17]. In many cases, uncertainties are partially instead of completely known, for example, they may take values on a bounded and closed set, that is, a compact set, as assumed in the present paper. Thirdly, perturbations may result from modeling errors or aging and inevitably exist in any realistic problem [18–20]. Finally, knowing the property of a perturbed system would be of great importance provided that the stability property of the nominal system is known, since investigating the properties of nominal system is usually much easier than studying that of perturbed system. Thus, in this paper, we explore the condition imposed on perturbations such that the perturbed system can preserve the stability property of the nominal system.

Note that there are many kinds of perturbations [21]. It was shown in [22] that, for a switched linear system with bounded or convergent perturbations, the perturbed system behavior is similar to the perturbation, provided that the nominal system is exponentially stable. Hereafter we focus only on the "vanishing" perturbation, namely, the perturbation is zero if the state is zero [23]. In [23, Lemma 9.1], it was proved that, for an exponentially stable delay-free nonlinear system, the perturbed system is also exponentially stable provided that the perturbation satisfies a linear growth bound with a sufficiently small coefficient. Clearly, it is important to extend this result to nonlinear systems with delays. Recently, the stability issue of discrete-time switched linear

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systems with time-varying delays and perturbations was investigated [24], which concluded that the perturbed system is exponentially stable if the nominal system is exponentially stable and the perturbation is small enough. Similar conclusion holds for switched homogeneous systems (of degree one) with delays [25]. However, for general (switched) nonlinear systems with delays, the problem remains unsolved. The task here is to explore if an upper bound of perturbation exists to guarantee the perturbed system preserving the exponential stability of the nominal system.

Technically, handling such a problem is not trivial. In [23, Lemma 9.1], the Lyapunov function method was used, which is based on the fact that a nonlinear delay-free system is exponentially stable if and only if there exists a Lyapunov function satisfying some constraints; however, a similar conclusion is unavailable even in the context of switched linear systems with delays. In other words, the converse Lyapunov theorem does not apply here. Intuitively, if we just consider evolution of the perturbed system of a nominal system on a finite set, one may claim the existence of the upper bound of perturbation with which the perturbed system and the nominal system have similar trajectory. However, as the considered set approaches infinity, the upper bound may approach zero to guarantee that these two systems have similar stability property. Therefore, alternative idea is required to fulfil our task. The idea used here can be briefly described as follows: First prove the existence of upper bound of perturbation on a finite set (with this bound, the trajectory of the perturbed system can be upper bounded by an exponential function with the same decay rate as the nominal system), and then, by means of mathematical inductive principle, extend the finite set to the right infinite set with the same upper bound. It is important to point out that the reason to use this method lies in the invalidity of the Lyapunov method in this context and that our method may be applied to similar situations.

It is well-known that switching signal is an important factor affecting the dynamics of switched systems. For example, given subsystems of switched systems, different signals may result in quite different stability properties [26]. Therefore, we try to consider several commonly used switching signals so that the obtained results can be applied more widely.

The main contribution of the paper lies in the following aspects: First, two conditions are proposed which impose a constraint on the perturbation so that the perturbed system may preserve the exponential stability of the nominal system, locally or globally, depending on the perturbation and the nominal system itself. Second, in the case of the perturbation is partially known, a tuning factor is introduced such that the "tuned" system can preserve the exponential stability of the nominal system.

The rest of this paper is organized as follows. Preliminaries and problems to be treated are presented in Section 2, main results are proposed in Section 3, and a numerical example is provided in Section 4. Finally, Section 5 concludes this paper.

Notation: A^{T} is the transpose of matrix A. For vectors $\boldsymbol{x},\boldsymbol{y}$, $\boldsymbol{x}\succ$ ($\succcurlyeq, \prec, \preccurlyeq$)$\boldsymbol{y}$ means that $\boldsymbol{x}$ is entrywise greater than (greater than or equal to, less than, less than or equal to) $\boldsymbol{y}$. These symbols can be applied to matrix in an obvious manner. $\mathbb{R}^{n\times m}$ denotes the set of all real matrices of $n\times m$-dimension and $\mathbb{R}^n=\mathbb{R}^{n\times 1}$. $\mathbb{R}^n_{0,+}=\{\boldsymbol{x}\in\mathbb{R}^n, \boldsymbol{x}\succcurlyeq\boldsymbol{0}\}$. $\mathbb{N}_0$ denotes the set of all nonnegative integers and $\mathbb{N}=\mathbb{N}_0\backslash\{0\}$. For any $q\in\mathbb{N}_0, \mathbb{N}_q=\{q,q+1,\dots\}$ and for any $m\in\mathbb{N}$, $\underline{m}=\{1,\dots,m\}$ and $\underline{m}_0=\underline{m}\cup\{0\}$. $|a|$ is the absolute value of a real number a, and $|\boldsymbol{x}|=[|x|_1,\dots,|x|_n]^{\mathrm{T}}$ with $\boldsymbol{x}=[x_1,\dots,x_n]^{\mathrm{T}}\in\mathbb{R}^n$. The symbol $\boldsymbol{0}$ is an n-dimensional zero vector. $\|\boldsymbol{x}\|$ is any norm of vector $\boldsymbol{x}\in\mathbb{R}^n$. For any $a>0$, $\overline{\mathcal{B}}_a=\{\boldsymbol{x}\in\mathbb{R}^n : \|\boldsymbol{x}\|\le a\}$. If $\boldsymbol{x}(s)$ is defined on the set $\{-d,\dots,a\}$ with $a\in\mathbb{N}_0$, then for any $k\in\{0,\dots,a\}$, $\boldsymbol{x}_k(\theta)=\boldsymbol{x}(k+\theta)$ for all $\theta\in\{-d,\dots,0\}$, $c\boldsymbol{x}_k=c\boldsymbol{x}(k+\theta)$ for $\theta\in\{-d,\dots,0\}$, $\|\boldsymbol{x}_k\|=\max_{s\in\{k-d,\dots,k\}}\{\|\boldsymbol{x}(s)\|\}$. Throughout this paper, the dimensions of matrices and vectors will not be explicitly mentioned if clear from context.

2. Problem statements and preliminaries

Consider the following switched system:

$$\begin{aligned} &\boldsymbol{x}(k+1)=\boldsymbol{f}_{\sigma(k)}\left(k,\boldsymbol{x}(k),\boldsymbol{x}(k-d_{\sigma(k)}(k)),u_{\sigma(k)}(k)\right), \quad k\ge k_0\\ &\boldsymbol{x}(k)=\boldsymbol{\varphi}(k), \quad k\in\{k_0-d,\dots,k_0\} \end{aligned} \tag{2.1}$$

where $k_0\in\mathbb{N}_0, \boldsymbol{x}(k)\in\mathbb{R}^n$ is the state, the map $\sigma:\mathbb{N}_{k_0}\to\underline{m}$ is a switching signal with m being the number of subsystems. It is always assumed that σ is with switching sequence $\{k_i\}_{i=0}^{\infty}$ satisfying $k_i\in\mathbb{N}_{k_0}, k_i>k_{i-1}(\forall i\in\mathbb{N})$ and $k_i\to\infty$ as $i\to\infty$. For each $l\in\underline{m}$, $u_l(k)\in\mathbb{U}_l$ ($\forall k\ge k_0$) represents uncertain parameters, where $\mathbb{U}_l$ is a specified compact set, delays $d_l(k)\in\{d_{1l},\dots,d_{2l}\}$ with $d_{1l}\in\mathbb{N}_0, d_{2l}\in\mathbb{N}_0$, and $d=\max_{l\in\underline{m}}\{d_{2l}\}$. Thus, $\boldsymbol{f}_l$ is a map from $\mathbb{N}_{k_0}\times\mathbb{R}^n\times\mathbb{R}^n\times\mathbb{U}_l$ into $\mathbb{R}^n$. For any k, the corresponding uncertainty $u_{\sigma(k)}(k)$ is clearly dependent on switching signal σ. Therefore, it is reasonable to define $u(k)$ to be the uncertainty at instant k, that is, $u(k)\in\mathbb{U}_{\sigma(k)}$, which means that $u(k)$ takes values on a compact set for any k. Let $\mathbb{U}$ be the set of admissible uncertainties. $\boldsymbol{\varphi}$ is an initial vector-valued function.

The following assumption is made for system (2.1):

Assumption 1. The map $\boldsymbol{f}_l$ is locally Lipschitz at origin in the second and third arguments, uniformly in the first and last ones. Moreover, $\boldsymbol{x}=\boldsymbol{0}$ is an isolated equilibrium point of each subsystem. More precisely, there exist positive scalars $L_1,\tilde{r}_1,\overline{r}_1$ such that

$$\begin{aligned} &\|\boldsymbol{f}_l(\cdot,\boldsymbol{x},\boldsymbol{y},\cdot)\|\le L_1\left\|\left[\boldsymbol{x}^{\mathrm{T}}\ \boldsymbol{y}^{\mathrm{T}}\right]^{\mathrm{T}}\right\|, \quad \forall\boldsymbol{x},\boldsymbol{y}\in\overline{\mathcal{B}}_{\tilde{r}_1}\\ &\boldsymbol{f}_l(\cdot,\boldsymbol{0},\boldsymbol{0},\cdot),\ \boldsymbol{f}_l(\cdot,\boldsymbol{x},\boldsymbol{y},\cdot)\ne\boldsymbol{0}, \quad \forall l\in\underline{m},\forall\boldsymbol{x},\boldsymbol{y}\in\overline{\mathcal{B}}_{\overline{r}_1},(\boldsymbol{x},\boldsymbol{y})\ne(\boldsymbol{0},\boldsymbol{0}) \end{aligned} \tag{2.2}$$

If $\tilde{r}_1=\overline{r}_1=+\infty$, then $\boldsymbol{f}_l$ is globally Lipschitz and $\boldsymbol{x}=\boldsymbol{0}$ is the unique equilibrium point of each subsystem.

Remark 1. Note that, in general, the local Lipschitz property $\boldsymbol{f}_l$ does not guarantee the existence and uniqueness of the solution to system (2.1). However, this condition is enough for us to discuss local exponential stability of system (2.1). The second condition in (2.2) means that (2.1) has an isolated equilibrium point, which in fact indicates that there is only a unique equilibrium point in the domain we considered. Similar remark can be made for Assumption 2.

With the assumption that $\boldsymbol{x}=\boldsymbol{0}$ is an isolated equilibrium point of system (2.1), hereafter when we speak of the stability property of system (2.1), we refer to that of the origin equilibrium point.

There are different kinds of switching signals some of which are defined below.

Definition 1 (*Liberzon [27]*). For switching signal $\sigma(k)$ and any $T>k\ge 0$, let $N_\sigma(T,k)$ be the switching numbers of σ on an open interval (k,T). $\sigma(k)$ is said to have average dwell time τ_a and "chatter-bound" N_0 if there exist two positive numbers N_0 and τ_a such that $N_\sigma(T,k)\le N_0+\frac{T-k}{\tau_a}$. A switching signal $\sigma(k)$ is said to be periodic if there exists a scalar $\kappa>1$ such that $\sigma(k+\kappa)=\sigma(k)$ holds for any $k\in\mathbb{N}_0$, such a minimal κ is called the period of $\sigma(k)$.

The following four classes of switching signals which are frequently encountered in literature will be considered:

- $\mathbb{S}_1=\{\sigma(k):\sigma(k)\text{ is an arbitrary switching signal}\}$.
- $\mathbb{S}_2(N_0,\tau_a)=\{\sigma(k):\sigma(k)\text{ has average dwell time }\tau_a\text{ and chatter bound }N_0\}$.
- $\mathbb{S}_3(\kappa)=\{\sigma(k):\sigma(k)\text{ has period }\kappa\}$.
- $\mathbb{S}_4(\tau_d)=\{\sigma(k):k_{i+1}-k_i\ge\tau_d\ge 2,\forall i\in\mathbb{N}_0\}$.

For $\mathbb{S}_4(\tau_d)$, if $\tau_d=1$, then $\mathbb{S}_4(1)$ is actually the set of arbitrary switching signals, that is, $\mathbb{S}_4(1)=\mathbb{S}_1$. Hereafter, it is always assumed that $\mathbb{S}$ is arbitrarily chosen from $\{\mathbb{S}_1,\mathbb{S}_2(N_0,\tau_a),$

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