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Research Article

Solutions of nonlinear constrained optimal control problems using quasilinearization and variational pseudospectral methods

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ABSTRACT

An alternative method for nonlinear constrained optimal control problems is developed in this paper. The proposed method converts the nonlinear optimal control problem into a sequence of constrained linear quadratic (LQ) optimal control problems using quasilinearization methods. And then we present a variational pseudospectral method based on dual variational principles and pseudospectral approximations in order to transform the constrained LQ problem into standard linear complementary problems (LCPs) which can be solved easily. The proposed method is highly efficient due to the benefits of qualsilineaization techniques and the sparse and symmetric properties of coefficient matrixes obtained by variational principles. And solutions of high precisions can be obtained with few time nodes and boundary conditions can be prescribed because of pseudospectral approximations. Besides, extra costate estimations are not required simply because this method is constructed by dual variational principles. Several numerical examples are simulated and comparisons between different methods are offered to demonstrate effectiveness and advantages of the proposed method.

1. Introduction

Various control methods and strategies like optimal control [1,2], robust control [3] and fuzzy control [4] were developed in the past several decades because of their wide applications. As a subset of control theories, the optimal control has attracted much attention because of the vigorous advancements and wide applications of optimizations [1,2,4–6]. Nonlinear optimal control problems subjected to continuous or pointwise constraints, which are imposed at every point along trajectories [1,2], have been studied by many researchers because the problems are widely encountered in engineering applications such as chemical engineering, aeronautics and astronautics. According to whether quasilinearization techniques are adopted, numerical methods for nonlinear constrained optimal control problems can be categorized as two groups obviously.

As a class of the first type, nonlinear constrained optimal control problems are solved directly. For numerical methods, unknown variables need to be discretized or parameterized firstly to convert continuous problems into discrete problems. Two parameterization techniques are often used: (i) function expansions [7–10] and (ii) interpolations [11–19]. As for function expansions, orthogonal function sets such as Chebyshev polynomials [7], Haar functions [8], hybrid functions [9] and piecewise-constant basis functions [10], were utilized as the base of function expansions to approximate unknown variables, and the time domains were not discretized. As for function interpolations, however, time domains are discretized firstly since the unknown variables are approximated by interpolations based on discrete time nodes. Thus the coefficients of interpolations are just the unknown variables at discrete nodes and then boundary conditions can be prescribed exactly, while the coefficients of each function in expansions have no explicit physical meanings. Thus, numerical methods based on interpolations such as finite elements methods [11–12], have attracted much attention in engineering applications because of the above advantages.

Based on global Lagrange interpolations, pseudospectral (PS) methods which yield exponent convergence for smooth problems were established to solve optimal control problems [13,14]. The PS methods are constructed by global interpolations based on orthogonal collocation points, which are obtained from the roots of orthogonal polynomials including Chebyshev [13] and Legendre polynomials [14], and/or linear combinations of orthogonal polynomials and their derivatives. However, the use of high-degree global polynomials is required for non-smooth problems since exponent convergence is lost, and then resulted nonlinear programming problems (NLP) for which the density grows quickly as a function of the number of NLP variables and it may be difficult to

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compute a solution when the NLP is sufficiently large [15,16]. To reduce the significant computation of global high-degree PS methods, *h* methods which based on local interpolations were developed [15–19]. Based on local PS methods and adaptation techniques, Rao and his coworkers [15,16] established adaptive *hp* methods for nonlinear constrained optimal control problems. Marsden and his coworkers [17] proposed structure-preserving *h* methods, i.e., discrete mechanics and optimal control (DMOC), where the state equations are discretized based on Lagrange–d'Alembert principle. The above *h* methods belong to direct methods [2] since optimal control problems were transferred into NLP, and then extra costate estimations are needed and the formulation of costate estimations may be highly dependent on the interpolation points [14]. Recently, Peng et al. [18,19] developed symplectic methods for nonlinear optimal control problems without path constraints based on dual variational principles and analogies with finite element methods. Thus, the symplectic methods are also *h* methods. Optimal control problems were transformed into nonlinear equations and then optimization tools are not required. More importantly, extra costate estimations are not involved and the Jacobian matrixes are symmetric due to the dual variational principles. More recently, the authors established a variational pseudospectral method for optimal control problems without path constraints based on the combination of dual variational principles and pseudospectral methods [20]. The variational pseudospectral method shows exponent convergence and high efficiency, and eliminates the costate oscillations in Lobbtao PS methods [14]. Therefore, nonlinear optimal control problems are converted into NLP or nonlinear equations and then solved directly based on various parameterization schemes in the above mentioned methods which belong to the first type methods.

As a class of the second type methods, however, nonlinear optimal control problems are converted into a sequence of quadratic optimal control problems or quadratic programming problems. It is well known that quasilinearization techniques can yield significant advantages compared with solving nonlinear problems directly [21–24]. The first advantage is the quadratic convergence, which is a most remarkable property for large-scale computations as computing times are usually directly proportional to the number of iterations and round-off errors can be increased seriously as the number of stages increases [21]. The second advantage is the monotonicity in the successive approximations which is an important property computationally [21]. Another advantage of quasilinearization is the robustness which means that algorithms do not require users to scale variables and constraints carefully and to offer very good initial guesses before starting calculations [22,23]. And then the quasilinearization is highly effective for optimization problems [23,24].

The quasilinearization methods have been successfully applied to nonlinear constrained control problems [25–28]. There are two possible quasilinearization methods to approximate a nonlinear constrained control problem by a sequence of quadratic problems according to whether or not the parameterization is implemented before the quasilinearization [25]. In the first case, the parameterization is implemented firstly and nonlinear optimal control problems are converted into NLP, and then quasilinearization techniques are applied to NLP [26]. As for the second case, quasilinearzations are conducted firstly and then nonlinear optimal control problems are transferred into a series of constrained linear quadratic (LQ) optimal problems [25,27,28]. As for the parameterization methods to discrete the constrained LQ optimal control problems, Jaddu used the function expansion based on Chebyshev polynomials [25,27], and then LQ optimal control problems were formulated as quadratic programming problems which are solved by optimization tools. Meanwhile, in the symplectic sequence iteration methods developed recently [28], local Lagrange interpolation methods were used and the constrained LQ problems were converted into standard linear complementary problems (LCPs) based on dual variational principles and complementary conditions. The coefficient matrixes are sparse and symmetric and no extra estimations are involved in symplectic methods because they are constructed using variational principles. However, there is a limitation in the derivation of our previous work [28]. One may find that the parametric variables were assumed to be constants in each subinterval in the derivation of the symplectic method [28], and then costate variables are linearly interpolated and state variables were constants within subintervals to match the interpolation of parametric variables in computations. Thus, unknown variables were approximated by local interpolations with very low order within subintervals and we need to increase the number of subintervals to a large value to reach desirable accuracies. Thus, variational methods with interpolations of high order need to be further developed.

In order to combine the advantages of interpolation parameterizations, quasilinearization methods, variational principles and PS methods, we will develop a variational pseudospectral iteration method for nonlinear constrained optimal control problems in this paper. In the proposed method, the nonlinear constrained optimal control problem is converted into a sequence of constrained LQ optimal control problems and then a variational pseudospectral method is developed to solve constrained LQ optimal control problems. In the proposed variational pseudospectral method, the unknown variables including state, control and parametric variables will be approximated by global interpolations based on Legendre–Gauss–Lobatto points, and then the constrained LQ optimal will be transferred into standard LCPs based on dual variational principles and complementary conditions. The proposed method has several advantages:

(i) The boundary conditions can be exactly imposed in advance since it is discretized by interpolations of Legendre–Gauss–Lobatto points. By contrast, approximations based on function expansions [7–10] and interpolations by Legendre–Gauss points [14] or Legendre–Gauss–Radau points [15,16] need to impose boundary conditions extraly;

(ii) It is noted that the introduction of quasilinearization approaches can bring about benefits like quadratic convergence, monotonicity and robustness [21–24]. As a result, the proposed method has good performance in terms of convergence and robustness in iteration processes due to the application of quasilinearization techniques;

(iii) The coefficient matrixes involved in the proposed method is symmetric and sparse, and no extra costate estimations are required as it is based on variational principles. And then the proposed method is highly efficient since few memory spaces are needed. On the contrary, the coefficients matrixes of collocation methods like direct PS methods are not symmetric and the density grows quickly as a function of the number of NLP variables [15,16], which result in costly computational burden;

(iv) Numerical solutions of high precision can be obtained with few discrete time nodes because the order of the interpolation of unknown variables such as state and parametric variables is much higher compared with the symplectic iteration method [28].

In the numerical example section, several examples will be simulated and comparison between different methods will be presented to demonstrate the above mentioned advantages.

The rest of this paper is organized as follows. In Section 2, the formulation of nonlinear constrained optimal control problems will be presented. Then, the application of quasilinearization methods to convert the constrained nonlinear optimal control problems into a series of constrained LQ problems will be given in Section 3. In Section 4, variational pseudospectral methods to formulate constrained LQ problems as standard LCPs will be

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