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Research article

Continuous higher-order sliding mode control with time-varying gain for a class of uncertain nonlinear systems

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A B S T R A C T

This paper presents a continuous higher-order sliding mode (HOSM) control scheme with time-varying gain for a class of uncertain nonlinear systems. The proposed controller is derived from the concept of geometric homogeneity and super-twisting algorithm, and includes two parts, the first part of which achieves smooth finite time stabilization of pure integrator chains. The second part conquers the twice differentiable uncertainty and realizes system robustness by employing super-twisting algorithm. Particularly, time-varying switching control gain is constructed to reduce the switching control action magnitude to the minimum possible value while keeping the property of finite time convergence. Examples concerning the perturbed triple integrator chains and excitation control for single-machine infinite bus power system are simulated respectively to demonstrate the effectiveness and applicability of the proposed approach.

1. Introduction

Sliding mode control (SMC) has been regarded as a powerful method to design robust controllers for uncertain nonlinear systems [1]. However, unexpected chattering phenomenon is produced by high-frequency switching control, which is a key issue when sliding mode controller is implemented practically. In addition, the conventional SMC demands that system relative degree with respect to sliding variable is 1, i.e., the control has to explicitly appear in its first total time derivative [2].

High-order sliding mode (HOSM) control, which can achieve better accuracy than the conventional SMC, has recently been proposed to reduce the chattering effect, owing to the application of discontinuous sign-function on high-order time derivative of sliding variable and eventually on time derivative of control input [3–6]. So far, it has found wide application in areas of hybrid vehicle [7], induction motor [8], flexible hypersonic vehicle [9], electric power system [10], and so on.

As a special case of HOSM control methods, second-order sliding mode algorithms can cope with nonlinear systems with relative degrees 1 or 2 [11–13]. Particularly, super-twisting algorithm has been an effective way of realizing second-order sliding mode, since it only requires measurement of sliding variable without using information about derivatives of sliding constraint. Its main difference from the conventional first-order sliding mode is that discontinuity appears in the second derivative of the switching function, whereas a non-Lipschitz term appears in the first derivative to achieve finite-time convergence [14–16]. Therefore, continuous control function is generated to drive sliding variable and its derivative to zero in finite time in the presence of smooth matched disturbances with bounded gradient.

More generally, arbitrary-order sliding mode control algorithms for uncertain affine SISO nonlinear systems have been proposed recently [17–19]. These algorithms allow the tracking of smooth signals by tuning only one gain parameter. Yet there is no constructive condition for the gain tuning, which has to be chosen sufficiently large to achieve finite-time stabilization. Levant [20] proposes a common method for the proper control gain adjustment based on the homogeneity approach. The result allows increasing or reducing the gain function, which logically leads to the idea of diminishing or even zeroing the control. Yet, in this case the ideal accuracy is inevitably lost. In addition, for these algorithms [17–20], the items $f(x,t)$ and $g(x,t)$ in the rth total time derivative of sliding variable $\sigma(x,t)$ ($\sigma^{(r)}(x,t)=f(x,t)+g(x,t)u$) are totally assumed to be some uncertain functions which further complicates the parameter adjustment.

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Actually, the $f(x,t)$ and $g(x,t)$ in most of uncertain affine nonlinear systems can be divided into two parts, i.e., the nominal part known in advance, and the uncertain part. Hence, a new group of HOSM controllers are proposed [21–27]. The HOSM control problem of uncertain affine nonlinear SISO systems is firstly formulated as the input–output terms by means of the differentiation of the sliding variables, which is equivalent to the finite-time stabilization of integrator chain system with bounded uncertainties. Then one part of the auxiliary feedback control law gets finite time convergence of nominal system, and the other part achieves system robustness. In these schemes [21–27], the realization of robustness is based on conventional first-order sliding mode, making the whole control input discontinuous and the chattering phenomenon obvious. Though artificially increasing the relative degree one can make control input continuous and reduce (or remove) chattering, more information or higher-order differentiation is needed in that case, inevitably increasing the system sensitivity with respect to the measurement noises.

Thus, the super-twisting second-order algorithm could be employed to substitute the first-order SMC part [21–27], and acts as the robust item to achieve control continuity and reduce chattering. However, uncertainties are generally time-varying in practice which causes chattering amplitude to be rather conservative, and sometimes the uncertain upper-bound is unknown, rendering invalidation of fixed-gain super-twisting algorithm [14]. Meanwhile, it is well known that the chattering amplitude is proportional to the magnitude of switching control gain [28]. As a consequence, time-varying gain SMC, in which the gain is adjusted according to an actual bound of the uncertainty, can reduce the chattering amplitude of control action to the minimum possible value, and compensate perturbation whose bound is state dependent or can only be calculated on-line.

Some attempts [23–27] have been made to constitute time-varying gain HOSM controllers, but time-varying characteristics only have effect on the control gain of first-order sliding mode, which makes the whole HOSM control laws still discontinuous. It will be more chattering reducing efficient by incorporating time-varying control gain with the continuous HOSM method. To date, papers referring to continuous HOSM with time-varying gain are quite limited, except the above mentioned method of raising system orders artificially. A few of the methods without demanding higher system orders mainly focus on adaptive gain continuous second-order SMC for system with relative degree 1. Paper [16] presents a super-twisting adaptive sliding mode control law. Yet, the constraint conditions on uncertainties are excessively rigorous. In [29], a class of smooth second-order sliding mode control laws for first-order dynamic systems are proposed. Nevertheless, the form of disturbances is rigorously assigned. An equivalent control based adaptive super-twisting algorithm is proposed in [28] for nonlinear system with relative degree 1. It enables reducing the control action magnitude to the minimum possible value while keeping the property of finite time convergence.

Consequently, the objective of this paper is to propose time-varying gain continuous HOSM control scheme for uncertain affine nonlinear SISO systems with arbitrary relative degree. Firstly, HOSM control is taken as finite time stabilization of perturbed integrator chains. Then the homogeneous continuous control part realizes finite time convergence of nominal system, and effectively adjusts transient time. In view of equivalent control concept, super-twisting algorithm is employed to eliminate the harmful effect of twice differentiable uncertainty and guarantee system robustness. Time-varying control gain is constructed to get minimum admissible value of the switching control gain amplitude. The whole control process is continuous, and then the control chattering is greatly decreased. The finite time convergence to HOSM is formally proved. Numerical example verifies the effectiveness and superiority, and eventually the proposed approach is applied to enhance transient stability for single-machine infinite bus power system.

The remainder of this work is organized as follows. Section 2 states the problem of HOSM control for an uncertain nonlinear system as the stabilization of a perturbed integrator chains. In Section 3, the proposed time-varying gain continuous HOSM control is designed and finite time stability is formally proved. Section 4 presents the simulation application of perturbed triple integrator chains and excitation control for single-machine infinite bus power system. Finally, some concluding remarks are given in Section 5.

2. Problem statement

Consider the following uncertain affine nonlinear system

$$\begin{cases} \dot{x} = a(x,t) + b(x,t)u \\ y = \sigma(x,t) \end{cases} \tag{1}$$

where $x \in R^n$, $u \in R$ are state variable and control input respectively, $\sigma(x,t)$ is the measurable smooth output vector known as sliding variable, $a(x,t)$ and $b(x,t)$ are smooth uncertain functions including internal parameter perturbations and external disturbances.

Assumption 1. The relative degree of system (1) with respect to $\sigma(x,t)$ is constant and known, and the associated zero dynamics are stable.

The HOSM control method allows finite time stabilization of $\sigma(x,t)$ together with its $r-1$ first time derivatives by constructing an appropriate feedback control law. The rth time derivative of $\sigma(x,t)$yields

$$\sigma^{(r)}(x,t) = f(x,t) + g(x,t)u \tag{2}$$

with $f(x,t) = L_a^r \sigma(x) = \sigma^{(r)}(x)|_{u=0}$, $g(x,t) = L_b L_a^{(r-1)} \sigma(x,t)$.

Then rth-order sliding mode control of system (1) with respect to sliding variable $\sigma(x,t)$ can be equivalently expressed as finite time stabilization of the following uncertain system

$$\begin{cases} \dot{z}_1 = z_2 \\ \quad \vdots \\ \dot{z}_{r-1} = z_r \\ \dot{z}_r = f(x,t) + g(x,t)u \end{cases} \tag{3}$$

where $z = [z_1, z_2, \ldots, z_r]^{\mathrm{T}} = \left[\sigma(x,t), \sigma^{(r-1)}(x,t)\right]^{\mathrm{T}}$.

It is supposed that system(3) can be divided into the nominal part which is known in advance, and the uncertain part [26], such that it can be rephrased as

$$\begin{cases} \dot{z}_1 = z_2 \\ \quad \vdots \\ \dot{z}_{r-1} = z_r \\ \dot{z}_r = f_0(x,t) + g_0(x,t)u + d(x,t) \end{cases} \tag{4}$$

where $f_0(x,t)$, $g_0(x,t)$ are known items, $d(x,t) = \Delta f(x,t) + \Delta g(x,t)u$ represents the lumped uncertainty, and it is assumed that $g_0(x,t)$is non-singular.

Assumption 2. There exists second-order differential of $d(x,t)$.

Consider static feedback control for the system (4)

$$u = \frac{1}{g_0(x,t)}(-f_0(x,t) + \tau) \tag{5}$$

where τ is the auxiliary control input. This feedback control realizes linearization of system(4) when $d(x,t) = 0$.

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