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Computational methods for the fast boundary stabilization of flexible structures. Part 1: The case of beams

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Abstract

An efficient active control strategy for flexible systems [V. Komornik, Rapid boundary stabilization of linear distributed systems, SIAM J. Control Optim. 35 (5) (1997) 1591–1613] is thoroughly investigated from the numerical point of view. A non-standard computational framework is proved to be relevant both for simulation and control synthesis. The proposed formulation proves necessary in so far as a standard numerical approach is shown to fail. The observed properties of the state feedback law confirm the theory as far as beams are concerned. In particular, one can achieve an arbitrarily large decay rate of some weak norm of the system. Moreover, the control law compares favourably with the integral force feedback in terms of efficiency. Finally, smoothing procedures are introduced to decrease the control spill-over associated with the possible lack of compatibility between boundary control and initial conditions. This purely artificial spill-over is proved to result from oversimplification of the control process modelling.

Keywords: Boundary control; Very weak formulation; Closed-loop stability; Fast stabilization; Spill-over

1. Introduction

There exist various strategies to control the vibrations of flexible structures: the most natural one is probably the direct velocity feedback (DVF) [6] that amounts to impose a force opposite to the measured velocity at the same point. The integral force feedback (IFF) [1,62] can be viewed as a special implementation of the DVF with built-in roll-off at high frequency. These collocated approaches do not rely on any model, and thus do not generate spill-over, but may lack efficiency due to the possible spatial localization of the damping effect and to the necessary low- or high-pass filtering which may deteriorate the performances at high or low frequencies. Moreover, spill-over may show up in practice due to the difficulty to achieve collocation, and due to sensor or actuator dynamics. Non-collocated approaches such as the classical LQ or H^{∞} strategy are *a priori* more efficient provided that controllability hold [71,65,58,31,45]. Nevertheless, a state-space model is required for implementation purposes, and its necessary finite-dimensional approximation may give rise to spill-over instabilities [5], although several large-scale field implementations on bridge piles during construction are based on non-collocated strategies [39].

Following Lions works [50–52] on the exact controllability of partial differential equations, Komornik has introduced an efficient and simple strategy to stabilize linear evolution equations including the case of flexible structures [40–42]. His method assumes the exact controllability of the system to hold at the continuous level. This property depends on the physics of the system and of the geometry. See *e.g.* [52,10,43,63,37,48,30,32,3,55,46,47,54,70,25,38,11,15,18]. Whenever the exact controllability of the structure holds, just inverting a modified controllability Gramian [40–42] leads to a control ensuring a uniform decay rate

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of the total energy which is larger than some parameter ω that can be arbitrarily chosen in the design of the control law. Recalling the difficulty to predict the decay rate of the energy of the closed-loop system for a given control law [43,12,42], it is worth emphasizing that here not only you can predict it but you can even impose *a priori* that some suitable norm of the state be bounded by $C(\omega)e^{-\omega t}$ at any time t, for any value of the design parameter ω. Moreover this property holds uniformly with the modal discretization [17] at the semi-discrete level. This control algorithm may be considered as a special case of pole placement algorithm, although the poles of the closed-loop system are not specified during the control synthesis. It also inherits the advantages of LQG–LTR approaches since the control law is optimal for a non-standard criterium that strongly involves the characteristics of the system to be controlled and of the actuators.

But in view of investigating the behavior of the control law numerically, we need a method of approximation both for the computation of the control and for the simulation of the controlled system. A major outcome of this paper can be stated as follows: consider a structure controlled by means of a displacement imposed on the boundary. The standard way of extracting the pseudo-dynamical part from the full response in order to solve a homogeneous evolution equation leads to a discrete system which is always unstable no matter the static linear control law (the existence of a stabilizing dynamic feedback is proved in [20])! This is in fact the main motivation for introducing an *ad hoc* formulation for boundary control.

Besides, it was proved elsewhere [17] that a so-called *very weak in space* formulation [18] leads to a discrete system that inherits the stability of the continuous system controlled with Komornik's algorithm. We show here that the same formulation also proves useful for a general LQR strategy as well.

Therefore the purpose of this contribution is twofold: first we promote a computational framework both for control synthesis and simulation of flexible systems controlled from the boundary. In particular this paper improves on the relevance of the *very weak in space* formulation used for the simulation (Section 3), and describes a new high-level "black-box" implementation of the proposed feedback. The relevance of both an unusual formulation for the dynamics of structures and a two-layer approximation of the controller equation is substantiated by numerical evidence in the case of a closed-loop system. The computational framework differs from [34,45].

A formal and general way of deriving the new formulation from a more classical one at the discrete level is also shown.

Second we assess numerically the efficiency of Komornik's feedback *per se* and in comparison with the IFF strategy.

Third, we tackle the control spill-over strictly associated with the possible lack of compatibility between boundary control and initial conditions. This kind of spill-over does not show up in practice and can be viewed as a purely numerical artifact resulting from the oversimplification of the control process modelling.

This paper builds upon and extends several existing contributions [21,22,17,19] which are scattered in conference proceedings.

This paper is organized as follows: Section 2 is devoted to rephrasing Komornik's control law in a suitable way in view of computations. A simple Euler–Bernoulli–Navier beam controlled at one end point, and an abstract system in state-space form are considered. The computational aspects of the theory are detailed in Section 3, at least for beams. Section 4 describes various numerical tests proving the efficiency of the numerical methods and control algorithm. The question of spill-over is addressed in Section 5, where two simple smoothing techniques are explained.

Finally, concluding remarks can be found in Section 6.

2. Rapid stabilization algorithm

This section owes much to [42]. However, the corresponding control algorithms are expressed here in a variational setting in view of computations. Moreover, the introduction of a fictitious time s for the control synthesis in addition to the real time t for the controlled system seems to be useful.

2.1. Euler–Navier–Bernoulli beams

Let us consider a simply supported Euler–Navier–Bernoulli beam of length L (see Fig. 1). Let ρ, E, A, I denote its mass density, Young's modulus, cross-sectional area and inertia respectively. For the sake of simplicity, the mass density per unit length ρA and the stiffness EI are supposed to be constant along the beam. Moreover, damping is not taken into account. Besides, many real-life structures such as cables have no damping or even a negative damping in case of adverse wind–structure interaction. Therefore it is fair to assess the effectiveness of control laws without the help of any expected preexisting damping. However, damping would comply with the theory [68,69] provided observability holds and damping is not too large. It is not clear that damping will always improve the performance of the controller.

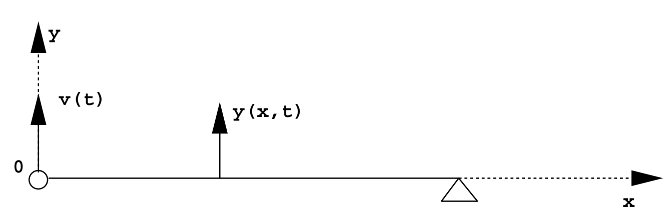

Fig. 1. The mechanical system.