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Research Article

Preference adjustable multi-objective NMPC: An unreachable prioritized point tracking method

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ABSTRACT

This paper proposes a new preference adjustable multi-objective model predictive control (PA-MOMPC) law for constrained nonlinear systems. With this control law, a reasonable prioritized optimal solution can be directly derived without constructing the Pareto front by solving a minimal optimization problem, which is a novel development of recently proposed utopia tracking approaches by additionally considering objective preferences with more flexible terminal and stability constraints. The tracking point of the proposed PA-MOMPC law is represented by a parametric vector with the parameters adjustable on the basis of objective preferences. The main result of this paper is that the solution obtained through the proposed PA-MOMPC law is demonstrated to have two important properties. One is the inherent Pareto optimality, and the other is the priority consistency between the solution and the tuning parametric vector. This combination makes the objective priorities tuning process transparent and efficient. The proposed PA-MOMPC law is supported by feasibility analyses, proof of nominal stability, and a numerical case study.

1. Introduction

Multi-objective nonlinear model predictive control (NMPC) simultaneously handles the optimization problem of multiple conflicting performance criteria over a receding horizon for constrained nonlinear systems. In industry, reconciling multiple objectives of tracking or economic performance and sustainability is crucial for some energy and chemical systems [1]. Particularly, there are some situations where some objectives are more important than others. Taking power generation systems as an example, the objectives related to pollutant emissions should have higher priorities over others especially when an extremely tight environmental policy is implemented by the government. Therefore, taking priorities into account is an important technical issue for multi-objective model predictive control (MOMPC) problems.

The optimization of MOMPC schemes relies on multi-objective optimization (MOO) algorithms. According to [2,3], traditional priority related MOO approaches include the weighted sum approach [4], the goal attainment method [5], and the lexicographic method [6], etc. However, these MOO methods have some limitations in handling priorities. The first two aforementioned MOO methods have no clear correspondence between weights and priorities of different objectives and thus a reasonable prioritized solution can only be obtained through tedious trial and error procedures, whereas the lexicographic method is completely dependent on priority assignments and normally derives undesirable results for objectives with lower priorities [2]. In the past decade, these priority related MOO approaches have been further studied and used to formulate different MOMPC controllers (see [7–10] and the references therein). For instance, the weighted sum method was further explored in [7] to obtain a scalar objective function, and a novel time-varying and state-dependent reference weight vector was proposed as an additional tuning parameter to handle priorities and guarantee closed-loop stability for the MOMPC scheme. In [8,9,11], the lexicographic method was further studied to handle priorities in the MOMPC scheme. Among them, theoretical results of feasibility and stability of the lexicographic method dealing with MOMPC problems were reported in [9]; however, relevant theoretical analyses of feasibility and stability for changing priorities formulations are still open issues.

Another technical issue for most MOMPC problems is the computational burden of constructing the Pareto optimal set before selecting a preferred solution based on expert knowledge. Recent advances w.r.t. MOMPC formulations have been reported in [1,3,7,10,12–14]. Among them, the utopia-tracking MPC (UT-MPC) framework originally proposed in [1] is a prospective option. In addition to computational burden reduction, it can also handle

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different types of performance criteria such as tracking, regulation and economic stage costs, etc. The key idea of this MOMPC formulation is to minimize the distance of the objective functions vector to a steady-state utopia point, which is the intersection of independently minimized objectives subject to certain constraints. A Pareto optimal compromise solution can be automatically obtained in this UT-MPC framework without constructing the entire Pareto optimal set; meanwhile, the nominal stability of the closed-loop system is guaranteed by a strong duality assumption and a steady-state terminal equality constraint. Inspired by the fundamental work of [1], significant progress w.r.t UT-MPC studies has been reported recently. In [13], a dynamic multiple objective optimization problem for cyclic processes was considered in the UT-MPC framework. In this work, the recursive feasibility of the dynamic UT-MPC scheme was analyzed, but the closed-loop stability proof was not provided. In [14], a dual-mode tracking strategy was implemented in the UT-MPC framework, which enlarged the region of attraction and further reduced the computational burden of the original UT-MPC problem.

However, existing UT-MPC approaches do not fully consider objectives preferences. The Pareto optimal solution (compromise point) derived by available UT-MPC approaches may not always reflect objective preferences [10]. To overcome this shortcoming, the current work proposes a new preference adjustable MOMPC (PA-MOMPC) scheme. Compared with existing UT-MPC approaches, the proposed PA-MOMPC approach has three more tuning parameters: the first one is the tracking point, which is represented by a parametric vector with the parameters adjustable on the basis of objective preferences w.r.t. system requirements (e.g. economic profits, environmental issues, etc.); the second one is within the terminal constraint, which is represented by an offset cost function with a tunable weighting parameter to maintain system dynamic performances; the third is within the stability constraint, which is a new stability perspective recently proposed by [15] to be more flexible than previous stability related assumptions in most UT-MPC approaches. With the proposed PA-MOMPC law, a reasonable prioritized optimal solution (referred to in this paper as prioritized compromise solution) can be directly derived without constructing the Pareto front by solving a minimal optimization problem, which is mathematically formulated as a distance criterion between the objective functions vector and the tunable tracking point in the objective functions space. The main result of this paper is that the solution obtained through the proposed PA-MOMPC law is demonstrated to have two important properties. One is the inherent Pareto optimality, and the other is the priority consistency between the solution and the tuning parametric vector. This combination makes the objective priorities tuning process transparent and efficient. On the other hand, due to the new tracking strategy and terminal conditions, the proposed PA-MOMPC formulation is different from the UT-MPC approach [1] in both feasibility analysis and stability proof. For this reason, the relevant analysis of feasibility and proof of stability are included in this paper to better support the proposed PA-MOMPC law.

The paper is organized as follows. Section 2 presents basic concepts of the MOO problems and demonstrates the Pareto optimality and priority of the prioritized compromise solution obtained through the preference adjustable MOO approach. Section 3 gives the formulation of the PA-MOMPC law and the feasibility analysis and the closed-loop stability of the proposed controller. The proposed PA-MOMPC method is illustrated by a numerical case study in Section 4. Conclusions are given in Section 5.

2. Preliminaries and problem statement

In this section, we first give some basic concepts for the multi-objective optimization (MOO) problem as well as the utopia-tracking (UT) framework. The preference adjustable (PA) MOO formulation is introduced at the end of this section.

2.1. Preliminaries

We define the notations as follows:

$\mathbb{R}:=(-\infty,+\infty)$; $\mathbb{R}^+:=\{r\in\mathbb{R}; r>0\}$; $\mathbb{N}:=\{1,2,3,\ldots\}$;

$\mathbb{Z}_{\geq 0}:=\mathbb{N}\cup\{0\}$; $\mathbb{Z}_{a:b}=\{z\in\mathbb{N}: z\geq a, z\leq b; a,b\in\mathbb{Z}_{\geq 0}\}$.

We consider a system described by a constrained discrete-time model

$$x_{k+1}=f(x_k,u_k), \tag{1}$$

where $x_k\in\mathbb{R}^{n_x}$ and $u_k\in\mathbb{R}^{n_u}$ are the state and control vector of the system at time step $k\in\mathbb{N}$. The states and controls are under constraints

$$x_k\in\mathbb{X}, u_k\in\mathbb{U},\quad \forall k\in\mathbb{N}, \tag{2}$$

where $\mathbb{X}\subseteq\mathbb{R}^{n_x}$ and $\mathbb{U}\subseteq\mathbb{R}^{n_u}$ are assumed to be compact sets. We also assume that $f(\cdot)$ is Lipschitz continuous in both arguments with a constant $L_f\geq 0$, and it satisfies $f(x^s,u^s)=x^s$ at an equilibrium point (x^s,u^s), which exists in the sets $\mathbb{X}\times\mathbb{U}$. The set of admissible equilibrium states is defined as follows:

$$\mathbb{D}_s:=\{(x,u)\,|\,x\in\mathbb{X}, u\in\mathbb{U} \text{ and } x=f(x,u)\}. \tag{3}$$

$$\mathbb{X}_s:=\{x\in\mathbb{X}\,|\,\exists u\in\mathbb{U} \text{ such that } x=f(x,u)\}. \tag{4}$$

The notation $\{x_k,u_k\}_b^{b+N}$ is used to describe a trajectory $(x_k,u_k), k\in\mathbb{Z}_{b:b+N}$ computed at the current time instant b, and N is the predictive horizon of the trajectory.

2.2. Basic steady-state multi-objective optimization

We define the steady-state multi-objective optimization problem as

$$\min_{x,u}[J_1(x,u),J_2(x,u),\ldots,J_M(x,u)] \tag{5a}$$

$$\text{s.t. } x=f(x,u),\quad x\in\mathbb{X},\ u\in\mathbb{U}. \tag{5b}$$

Here, we assume that the cost functions or performance indices $J_i(\cdot,\cdot):\mathbb{R}^{n_x\times n_u}\longrightarrow\mathbb{R}, i\in\mathbb{Z}_{1:M}$ are Lipschitz continuous in both arguments with corresponding constants $L_{J_i}, i\in\mathbb{Z}_{1:M}$. The performance indices vector is defined as $J(\cdot,\cdot)^T:=[J_1(\cdot,\cdot),J_2(\cdot,\cdot),\ldots,J_M(\cdot,\cdot)]^T\in\mathbb{R}^M$, where $\mathbb{R}^M$ is the cost function space with M coordinates. The Lipschitz constants vector is denoted as $L_J:=[L_{J_1},L_{J_2},\ldots,L_{J_M}]^T\in\mathbb{R}^M$.

Unlike single-objective optimization problems, there does not exist a single global optimal solution for the MOO problems because of the conflicting nature between different objectives. Traditional MOO approaches generate a Pareto solution by first constructing the Pareto front and then selecting a fair point based on expert knowledge. The utopia-tracking approach proposed in [1] is straightforward and computationally efficient compared to traditional MOO approaches. It can directly derive a Pareto optimal solution without constructing the Pareto front.

Definition 1. (Steady-state Pareto optimal [2]). A steady-state feasible point $(x_c,u_c)\in\mathbb{X}\times\mathbb{U}$ for multi-objective optimization problem (5) is Pareto optimal iff there does not exist another