ARTICLE IN PRESS

ISA Transactions ■ (■■■■) ■■■–■■■

Contents lists available at ScienceDirect

ISA Transactions

journal homepage: www.elsevier.com/locate/isatrans

Set-membership methodology for model-based prognosis

Basma Yousfi [a,*], Tarek Raïssi [b], Messaoud Amairi [a], Mohamed Aoun [a]

[a] MACS Laboratory : Modeling, Analysis and Control of Systems LR16ES22, National Engineering School of Gabes (ENIG), University of Gabes, Tunisia.
[b] Conservatoire National des Arts et Métiers (CNAM), Cedric-Lab. 292, Rue St-Martin, 75141 Paris Cedex 03, France

ARTICLE INFO

Article history:
Received 24 February 2016
Received in revised form
28 August 2016
Accepted 29 September 2016
This paper was recommended for publication by He Peter.

Keywords:
Damage
Interval observers
Prognosis
Remaining useful life
Singularly perturbed systems

ABSTRACT

This paper addresses model-based prognosis to predict Remaining Useful Life (RUL) of a class of dynamical systems. The methodology is based on singular perturbed techniques to take into account the slow behavior of degradations. The full-order system is firstly decoupled into slow and fast subsystems. An interval observer is designed for both subsystems under the assumption that the measurement noise and the disturbances are bounded. Then, the degradation is modeled as a polynomial whose parameters are estimated using ellipsoid algorithms. Finally, the RUL is predicted based on an interval evaluation of the degradation model over a time horizon. A numerical example illustrates the proposed technique.

1. Introduction

One of the potential interests of prognosis is its scope, from an economic point of view, to the field of industrial maintenance. Maintenance activity combines various techniques, tools and methods to decrease maintenance costs while rising availability, reliability and security of equipments [1]. Various approaches to prognosis have been developed that range in fidelity from simple historical failure rate models to high-fidelity physics-based models. Indeed, prognosis approaches can be classified in three different types. The first one is based on experts knowledge [2,3], the second is based on data-driven [4,5] and the last type is characterized by model-based prognosis [6–12]. Actually, many applications, such as automotive, aerospace and defense industries, show a growing interest to model-based prognosis [1,11].

Usually, the dynamics of the degradation state are slower than that of the system state. Therefore, in this work, it is modeled as a slow dynamic state and the technique of singularly perturbed systems is chosen to model and estimate the degradation evolution. The singularly perturbed theory can highlight the decomposition of the system into various time scales (decoupling into slow and fast dynamics) as mentioned in [13–16]. It has been extensively studied in the literature and was used in filtering, estimation, control and systems theory [13,14]. Using that theory, the damage state can be estimated by an observer when only inputs and outputs measurements are available [6,17–19].

In the presence of uncertainty, design of a conventional estimator converging to the ideal value of the state cannot be realized. In this case an interval estimation may still remain feasible and an observer can be constructed that, using input–output information, evaluates the set of admissible values (interval) for the state at each instant of time. Interval observers are considered as an alternative technique for conventional robust state estimation. Initially, they have been used in the domain of biotechnological processes characterized by large uncertainties [20–22]. In the case of interval estimation, two conventional observers are designed to compute lower and upper bounds for the domain of the state vector and the width of the estimated interval depends of the model uncertainty. This approach is based on the monotone systems theory [23]. Several interval techniques for state estimation have been proposed in the literature (the reader can refer to [22,24–29,12,15,16] and the references therein). To the best of our knowledge, only few works have investigated model-based prognosis in a set-membership setting. In [30], the degradation is modeled as an unknown input which is estimated through an interval observer. The interval observer structure proposed in [30] is based on decoupling the unknown input effect on the state dynamics by solving algebraic constraints on the estimation errors. The main drawback is that the unknown input estimation requires the evaluation of the noisy measurements derivatives. To overcome such differentiation, we suggest in this paper a methodology

* Corresponding author.
E-mail addresses: yousfi.besma@gmail.com (B. Yousfi), tarek.raissi@cnam.fr (T. Raïssi), amairi.messaoud@ieee.org (M. Amairi), mohamed.aoun@enig.rnu.tn (M. Aoun).

http://dx.doi.org/10.1016/j.isatra.2016.09.025

based on singularly perturbed theory where the degradation is modeled as a slow dynamic state. An interval observer for singularly perturbed systems is developed.

The methodology proposed in this paper consists in using an interval observer to estimate the damage state based on the data available through measurements up to a current time instant. In addition, the degradation state estimated through the interval observer is used to identify the parameters of a polynomial that model the system degradation using the outer bounding ellipsoid strategy.

The methodology in this work is divided into two steps. The first one consists in building an interval observer to compute a lower and an upper bounds for the slow and fast subsystems after decoupling the full-order system using the singular perturbed approach. The interval observer design requires stability and cooperativity properties of the observation error that can be ensured through a judicious gain computation and changes of coordinates [27,31,25]. The second step consists in modeling the degradation as a polynomial and in predicting the remaining useful life.

The paper is organized as follows: Section 2 represents the problem statement. The main results for designing the interval observer are developed in Section 3. Section 4 is devoted to the prediction of the remaining useful life. Finally, some numerical simulations are given in Section 5 to illustrate the performances of the proposed methodology.

2. Problem statement

Prognosis can be viewed as an add-on capability to diagnosis that assesses the current health of a system and predict its remaining useful life based on sensed features that capture the gradual degradation in the operation of the system [11,32]. The degradation is a continuous variable whose evolution is described by a deterministic or stochastic process. The most common models are composed of difference, differential or of partial differential equations. Consider the degradation model of a system with the following form:

$$\begin{cases} \dot{x}_1 = f(x_1, \lambda(x_2), u) + \sigma_1 \\ \dot{x}_2 = \epsilon g(x_1, x_2, u) + \sigma_2 \\ y = h(x_1, x_2, u) + v \end{cases}, \tag{1}$$

where the state $x_1 \in \mathbb{R}^{n_1}$ is associated with the fast dynamic behavior of the system and the vector $x_2 \in \mathbb{R}^{n_2}$ represents the slow dynamic variables related to a system damage degradation. $u \in \mathbb{R}^m$ denotes the system input, the parameter vector $\lambda \in \mathbb{R}^q$ is a function of x_2, $y \in \mathbb{R}^p$ is the measured output, f, g and h are differentiable functions in adequate dimensions. The rate constant $0 < \epsilon \lll 1$ defines the time-scale separation between the fast dynamics and the slow drift. The disturbances $\sigma_1 \in \mathbb{R}^{n_1}$, $\sigma_2 \in \mathbb{R}^{n_2}$ and $v \in \mathbb{R}^p$ correspond respectively to the state and measurement noises which are assumed to be bounded with prior known bounds such that $|\sigma_1| \le \overline{\sigma}_1$, $|\sigma_2| \le \overline{\sigma}_2$, and $|v| \le V$, where $\overline{\sigma}_1 \in \mathbb{R}^{n_1}$, $\overline{\sigma}_2 \in \mathbb{R}^{n_2}$ and $V \in \mathbb{R}^p$ are constant componentwise positive vectors.

The model (1) is closely related to a standard singular perturbation model [13]

$$\begin{cases} \epsilon \dot{x}_1 = f(x_1, x_2, u) + \sigma_1 \\ \dot{x}_2 = g(x_1, x_2, u) + \sigma_2 \\ y = h(x_1, x_2, u) + v \end{cases}. \tag{2}$$

The only difference is that the dynamics of the fast-time process depend on x_2 instead of $\lambda(x_2)$, where $x_1 \in \mathbb{R}^{n_1}$ and $x_2 \in \mathbb{R}^{n_2}$ are respectively the fast and slow state variables. The global order is $n = n_1 + n_2$ and ϵ is a small positive number known as singular perturbed parameter. A reduced system is approximately obtained by formally setting $\epsilon = 0$ in (2) [33,34]

$$\begin{cases} 0 = f(x_{1_s}, x_s, u_s) + \sigma_1 \\ \dot{x}_s = g(x_{1_s}, x_s, u_s) + \sigma_2 \end{cases}, \tag{3}$$

where x_{1_s}, x_s and u_s represent respectively the slow components of the variables x_1, x_2 and u. By solving the first algebraic equation in (3), the solution $x_{1_s} = h(x_s, u_s, \sigma_1)$ is obtained and used in the second equation in (3) to rewrite it as [34–36]

$$\dot{x}_s = g(h(x_s, u_s, \sigma_1), x_s, u_s) + \sigma_2 = g(x_s, u_s, \sigma_1) + \sigma_2. \tag{4}$$

A boundary-layer system is defined by

$$\dot{x}_f(\tau) = f(x_f(\tau), x_2, u_f(\tau)) + \sigma_1(\tau), \tag{5}$$

where x_f is the fast component of x_1, u_f represents the fast component of u, $\tau = \frac{t}{\epsilon}$ is a stretching time scale and the vector x_2 is treated as a fixed parameter. For any positive ϵ sufficiently small, the actual states satisfy [37]

$$x_1(t) = x_{1_s}(t) + x_f\left(\frac{t}{\epsilon}\right) + O(\epsilon) \tag{6}$$

$$x_2(t) = x_s(t) + O(\epsilon), \tag{7}$$

for all t. Using Eq. (4) as the slow model and Eq. (5) as the fast one, one can anticipate to approximate x_1 and x_2 respectively by [36]

$$x_1(t) \cong x_{1_s}(t) + x_f\left(\frac{t}{\epsilon}\right), \tag{8}$$

and

$$x_2(t) \cong x_s(t). \tag{9}$$

Remark 1. The fast variables cannot always be explicitly expressed from (3). The most popular method used to deal with this problem is based on a change of coordinates [38,39] requiring a linear transformation in order to eliminate the fast dynamics.

Based on separate interval observers design for the slow and the fast subsystems, the paper presents a methodology to predict the remaining useful life of system (1) as shown in the next sections. In the sequel, the following notations are used:

- The set of real matrices with $n \times m$ elements is denoted by $\mathbb{M}_{n\times m}$, $I_n \in \mathbb{M}_{n\times n}$ depicts the identity matrix and E_p denotes the matrix with dimension $p \times 1$ with all elements equal 1.
- For two vectors $x_1, x_2 \in \mathbb{R}^n$ or matrices $A_1, A_2 \in \mathbb{M}_{n\times m}$, the relations $x_1 \le x_2$ and $A_1 \le A_2$ should be understood elementwise.
- For a measurable and locally essentially bounded input $u : \mathbb{R}_+ \to \mathbb{R}$, the symbol $\|u\|_{[t_0,t_1]}$ denotes its L_∞ norm : $\|u\|_{[t_0,t_1]} = ess\sup\{|u(t)|, t \in [t_0, t_1]\}$.
- The set of all inputs u with the property $\|u\| < \infty$ is denoted by $\mathcal{L}_\infty$.
- Given a matrix $A \in \mathbb{M}_{n\times m}$, define $A^+ = \max\{0, A\}$, $A^- = A^+ - A$ (similarly for vectors).
- The relation $P \prec 0$ $(P \succ 0)$ means that the matrix $P \in \mathbb{M}_{n\times n}$ is negative (positive) definite.
- For $\underline{x}, \overline{x} \in \mathbb{R}^n$, $[\underline{x}, \overline{x}]$ denotes the set $\{x \in \mathbb{R}^n \backslash \underline{x} \le x \le \overline{x}\}$.