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Research article

Robust stabilization of underactuated nonlinear systems: A fast terminal sliding mode approach

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ABSTRACT

This paper presents a fast terminal sliding mode based control design strategy for a class of uncertain underactuated nonlinear systems. Strategically, this development encompasses those electro-mechanical underactuated systems which can be transformed into the so-called regular form. The novelty of the proposed technique lies in the hierarchical development of a fast terminal sliding attractor design for the considered class. Having established sliding mode along the designed manifold, the close loop dynamics become finite time stable which, consequently, result in high precision. In addition, the adverse effects of the chattering phenomenon are reduced via strong reachability condition and the robustness of the system against uncertainties is confirmed theoretically. A simulation as well as experimental study of an inverted pendulum is presented to demonstrate the applicability of the proposed technique.

1. Introduction

One of the fascinating area, in the existing era, is the control of underactuated systems [1,2]. By definition, these systems are governed by fewer number of control inputs than the degree of freedom of the systems. In the context of importance, these systems are extensively used in the industry, for examples, underwater vehicles, air vehicles and humanoid robotics. Therefore, a wide number of researchers have devoted their efforts to acquire a sound mechanism for the control of such important class of systems. These systems, as outlined in [3], can not be stabilized by smooth feedback because the dynamics of this class are governed by differential equations in the presence of some non-integrable differential conditions [4,5]. Therefore, the control of this class is a challenging problem. Moreover, the control techniques vary from system to system for this class of dynamic systems and may not, in general, be applied to the entire class. In the existing literature, a wide number of approaches were used (see for instance, [7–14]) to stabilize this type of systems.

Sliding mode control approach (see for detail, [15,16]), which is famous for its robust nature against external and internal disturbances, was developed in [17,18] for the aforesaid class. The results developed by the aforementioned strategies were very interesting and were capable for handling a class of uncertainties. However, these techniques needed to have relative degree one (see for definition, [19]) of the system with respect to their proposed sliding manifolds. In addition, these techniques ensured asymptotic regulation, of the system's states, to the origin. Moreover, some of these algorithms needed the introduction of virtual disturbances to satisfy some conditions [17]. Some interesting results were presented in [20] for inverted pendulum and Furata pendulum, where coupled sliding surfaces based sliding mode strategy was devised. Very similar results, based on coupled sliding surfaces, were proposed in [22] and a terminal sliding strategy was proposed for the sliding mode enforcement. However, the coupled sliding surface based control laws become very complex in the presence of constraints related to each subsystem dynamics. In addition, often one needs the deliberate introduction of virtual disturbances in the system's dynamics which reduces the significance of these strategies. The existing literature also contains switching controller based strategies [23] which claims for finite time enforcement of sliding modes. However, the state convergence was asymptotic in nature which, consequently, resulted in low precision. The main drawback of this strategy is that if the hydrodynamics damping forces are nonlinear in nature then it will result in an indefinite sign of the derivative of the Lyapunov functions which results in the failure of the design procedure.

Currently, an output feedback second order sliding mode based strategy is presented in [21] with good simulation results but the

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output is regulated to the respective origin asymptotically which intern results in low precision. In addition, their procedure needs fourth order differentiation of the designed output which once again will result in substantial steady state error during experimental study.

In this work, the main objective is the finite time enforcement of sliding modes against a fast terminal attractor which will results in finite time stabilization of the system's states, suppression of the chattering phenomena and robustness in the presence of unmodeled dynamics which are ever demanding in the control of electro-mechanical systems. We argue that the contributions of this article are nontrivial in two major aspects. First, the hierarchical development of a fast terminal sliding manifold and the control law design for the underactuated electro-mechanical nonlinear systems. The proposed control law results in the enforcement of sliding mode, in finite time, against the designed fast terminal sliding manifold which, consequently, ensures finite time regulation of the system outputs to their desired points. Second contribution is the experimental validation of the aforesaid claims on the actual setup of an inverted pendulum. During the design, the underactuated system's dynamics, in the first step, are properly transformed into a regular form which subdivides the whole dynamics into two sub blocks. The first block is related to those states which are not directly affected by the control inputs. The second block, on the other hand, concerns with those dynamics which are directly driven by the control input. The indirectly controlled states are finite time stabilized by considering the directly controlled state as a virtual control input into the said block. The finite time stabilization results in more accuracy which is a clear benefit of our proposed strategy over the other techniques being reported earlier. Furthermore, the chattering across the switching manifold, which is strongly associated with sliding modes, is also suppressed/diminished. Note that, this newly suggested control methodology encompasses those underactuated nonlinear systems which can be transformed into regular form. One more aspect to highlight about the newly proposed methodology is that this strategy is not limited to underactuated systems. Any system transformable to regular form, controllable canonical forms, and normal forms can be controlled via our strategy in finite time. The rest of the paper contains: In Section 2 the problem statement is presented, Section 3 contains the general terminal attractor approach to this class of dynamic systems. In the same section, the stability of the proposed approach is presented. The cart pendulum example is considered as an illustrative example and their detailed simulation and experimental discussion is given is Section 4. The last section encompasses concludes the paper.

2. Problem description

The dynamic equations of any mechanical systems (particularly, underactuated systems), in general, are represented by a set of interconnected second order differential equations of the following form

$$J(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) + F(\dot{q}) = B(u + \delta(q,\dot{q},t)) \tag{1}$$

where $q \in R^n$ and $\dot{q} \in R^n$ are positions and velocities vectors which make a configuration space of $2n$ variables/states, respectively. $J(q) \in R^{n\times n}$ is the inertia matrix, $C(q,\dot{q}) \in R^{n\times n}$ are centrifugal and Coriolis forces, $G(q) \in R^{n\times 1}$ are gravitational forces, $F(\dot{q}) \in R^{n\times 1}$ are fractional forces and $B(q) \in R^{n\times m}$ is the control input channel. Furthermore, the controlled input $u \in R^m$ such that $m < n$. The term $B\delta(q,\dot{q})$ represents matched uncertainty due to unmodeled dynamics and external disturbances that is assumed norm bounded.

Remark 1. The proposed control design strategy can be employed to a wide class of nonlinear systems like systems in controllable canonical forms, output feedback linearizable systems i.e., normal forms and regular forms. Therefore, this newly proposed methodology is very significant in the nonlinear system's theory and can be used easily where finite time stabilization or asymptotic stabilization becomes the core objective.

Assumption 1. It is assumed that $J^{-1}(q)B$ is full rank matrix i.e.,

$$rank(J^{-1}(q)B) = m$$

Using nonlinear coordinate transformation of continuously differentiable functions, the system in (1), is transformed into the following set of second order differential equations the so-called regular form [24]

$$\ddot{E} = \Theta_1(E,\dot{E},H,\dot{H})$$
$$\ddot{H} = \Theta_2(E,\dot{E},H,\dot{H}) + \Phi(E,H)u + \Delta(E,\dot{E},H,\dot{H},t) \tag{2}$$

where $E \in R^{(n-m)}$ and $\dot{E} \in R^{(n-m)}$ represent, respectively, the position and velocity vectors of the system which is not directly affected the by applied control force u, $H \in R^m$ and $\dot{H} \in R^m$ are the position and velocity vectors of the system which are directly under the influence of the applied control force, $\Theta_2(E,\dot{E},H,\dot{H}) \in R^m$, $\Phi(E,H) \in R^m$ are smooth vector fields, and $\Delta(E,\dot{E},H,\dot{H},t)$ represents the matched uncertainties in the transformed form. In order to proceed smoothly to the fast terminal attractor (FTA) design, the following assumptions are furnished:

Assumption 2. It is assumed that the origin, in the state space of the system, is an equilibrium point of the open loop system. In other-words, $\Theta_1(0,0,0,0) = 0$ and $\Theta_2(0,0,0,0) = 0$

Assumption 3. The uncertainties term $\Delta(E,\dot{E},H,\dot{H})$ is assumed to be norm bounded by a positive constant κ i.e.,

$$|\Delta(E,\dot{E},H,\dot{H})| \le \kappa$$

Assumption 4. In order to meet the controllability condition, it is assumed that $\frac{\partial\Theta_1}{\partial\dot{H}}\Phi(E,H) \neq 0$.

The control objective is to regulate to the origin, in finite time, the states of the aforesaid two blocks in the presence of matched uncertainties.

3. Main results: terminal attractor and control design

In this section, the regular form (2) is taken into account and a step by step development, with different scenarios, is presented in a comprehensive manner. During the overall development the stability of the zero dynamics is kept at the top which lead to the forthcoming cases.

3.1. Case-1

The first subsystem in (2) does not depend directly on the control input while the second subsystem is directly driven by the control input u. Therefore, the first system works as internal dynamics. The zero dynamics of this system, with H as an output, can be obtained by substituting $H=0$ and $\dot{H}=0$ in the internal dynamics block i.e.,

$$\ddot{E} = \Theta_1(E,\dot{E},0,0) \tag{3}$$

If the system in (3) is stable asymptotically/exponentially, then, the only task left is to stabilize the second block in (2). The finite