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A level set method for topology optimization of heat conduction problem under multiple load cases

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Abstract

In this paper we present a numerical approach of topology optimization under multiple load cases for heat conduction problem. This framework is based on the theories of topological derivative and shape derivative for elliptic system. We employ level set model to implicitly represent geometric boundary of thermal conductive material. Introducing topological derivative will generate new topology in the design domain, which suppresses the dependence of initial topology guess to some extent. The shape optimization is obtained by combining shape derivative with level set method. The functional of quadratic temperature gradient is taken as the objective function in our analysis, which is subjected to the state equation of steady heat conduction and volume constrain. The shape of material domain is treated as the design variable and the final result is achieved by updating level set function gradually. We develop an effective numerical technique to implement the optimal design with multiple load cases for heat conduction problem. Numerical examples demonstrate that our proposed approach is effective and robust for topology optimization of heat conduction problem.

Keywords: Topological derivative; Shape derivative; Topology optimization; Level set method; Heat conduction; Multiple load cases

1. Introduction

Many structural topology optimization approaches have been introduced to the field of thermal structure design in recent years. Most of the studies are concentrated on the single load case and there are few literatures about the topology optimization of heat structure under multiple load cases. However, we often encounter that the multiple thermal loads or their different combinations act at different time in practical design problem of thermal structure.

Xie and Steven developed topology optimization of elastic structure with multiple load cases [1] by ESO (Evolutionary Structural Optimization) method [2]. The structural shape and topology is obtained by gradually removing the material region with low stress. Li et al. [3] investigated the shape and topology design of heat conduction structure using ESO method. They considered the optimization problem subjected to single heat load case and multiple heat load cases. The optimization process is realized by gradually removing a small part of conductive material, which has little contribution to the objective function. Neagu and Bejan [4] investigated constructal tree networks for heat transfer by constructal theory. They built conductive paths to decrease the heat resistance and enforce the dissipation of heat. Cheng et al. [5] employed the bionic optimization approach to build the effective heat transport paths. The main idea of bionic optimization method is gradually increasing high conductivity material where the temperature gradient is high, so the computational cost of the bionic method based optimization is expensive.

Another class of significant and popular approach for structural topology optimization is the homogenization method [6,7], which was developed by Bendsoe and Kikuchi [8]. Diaz and Bendsoe [9] implemented the shape optimization under multiple load cases by the

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homogenization method. The material model of the homogenization method is devised as multi-hole periodic microstructure cell. The material properties can be described by characteristics of microscopic unit. The topology optimization of structure is posed as the size changing of microstructure unit. However, the geometric description of microstructure needs plenty of variables and the intermediate densities of material elements often occur during the optimization process. The SIMP (Solid Isotropic Microstructure with Penalization) method as a variation and simplification of the homogenization method has many advantages and received wide consideration [10]. The densities of basic element are considered as the design variables. But the convergent solution of SIMP method strongly depends on the value of penalization term. The topology optimization of heat structure was implemented by the SIMP method [6]. Donoso and Sigmund [11] investigated the multiple physics design problem modeled by Possion's equation using topology optimization method. The optimization objective is to minimize the thermal resistance with optimal distribution of high conductivity material. However, both the homogenization method and the SIMP method have the problems of numerical instability such as checkerboards, mesh-dependencies and local minimal solutions [12]. Many numerical techniques have been studied for regulating the problems of numerical instability, such as filtering technique [13], perimeter method [14].

Level set method based topology optimization for elastic structure has been widely studied in recent years, since level set method can handle complex topology changes naturally and geometric characteristics can be computed by level set function. Sethian and Wiegmann [15] first introduced level set method for boundary design of elastic structure. They used level set model to describe the complex geometric boundaries of structure. The velocity of level set equation is described as a function of the stresses on the geometric boundary, and then the extension velocity is obtained by solving the partial differential equation on the reference domain. The addition and removal of material are determined by the evolving of level set function. The initialization of structure is furnished with multi-hole uniformly, and the level set method handles merging or splitting of holes easily during the optimization process. Osher and Santosa [16] investigated the shape optimization problems of structural dynamics using level set method. They took the eigenvalues and the area of material domain as the objective functions, respectively. They applied variational level set calculus and projected gradient method to build an effective numerical technique for shape optimization problems of vibration. Allaire and Jouve [17] combined the classical shape derivative theory with level set method for structural topology optimization. The topology optimization method coupled with level set model has been applied to linear and nonlinear elastic structure for two-dimensional and three-dimensional space problems. Structural compliance and the least square error compared to a given displacement were considered as the objective functions. They employed the adjoint method to implement shape sensitivity analysis and derived the velocity function of level set equation. They also applied the approach based on level set model to the structural design of vibration problem and multiple load cases [18]. Wangs [19–21] studied the topology optimization method based on level set model and extended the approach for the design of multiple materials using multi-phase level set model. Liu et al. [22] implemented level set model based structural topology optimization via software FEMLAB, which gave the procedure of incorporation between boundary propagation and finite element analysis software. Ha and Cho [23] used a weak variation form of heat conduction equation and variational level set calculus approach to find the velocity field of level set equation and achieved the optimal design of heat conduction problem. In order to obtain the right shape and topology of structure, the initial topology guess must be similar to the final result, which is the main disadvantage of topology optimization approach based on level set model.

Topological derivative method, which is considered as a direct method to obtain optimal shape and topology, has been applied to elastic structure, electromagnetic structure [24,25] and inverse problem [26] recently. Sokolowski and Zochowski [27,28] considered the topological derivative of arbitrary shape functional subjected to elliptic equation or elastic state equation. Topological derivative based topology optimization method has the ability to generate new holes by computing the sensitivity of the objective function about small hole in the design domain. They also provided the approach of calculating topological derivative and numerical examples to validate their conclusions. Novotny et al. [29] modified the definition of topological derivative and introduced a topological-shape sensitivity analysis approach. They presented a new method to compute the topological derivative, which was based on the theory of shape derivative. Burger et al. [30] proposed a method for shape reconstruction and optimization problem, which connected the topological derivative with level set method. Allaire et al. [31] employed the shape derivative, topological derivative and level set method for structural topology optimization. The topological derivative based method can create new holes during the topology optimization process and restrain the dependence of initialization with approximate topology guess.

In this paper we will investigate the numerical method of topology optimization under multiple load cases for steady heat conduction problem. Our goal of optimization is to construct the effective transport path for heat dissipation under a given volume constraint. We synthetically employ the shape derivative and topological derivative coupled with level set method for elliptic system. We investigate the models of shape derivative and topological derivative for multiple thermal load cases. The remainder of this paper is organized as follows. In Section 2, we give level set model for geometric description of heat-conductive structure. In Section 3, we present the objective function