ARTICLE IN PRESS

ISA Transactions ∎ (∎∎∎∎) ∎∎∎–∎∎∎

Contents lists available at ScienceDirect

ISA Transactions

journal homepage: www.elsevier.com/locate/isatrans

Research article

Adaptive arrival cost update for improving Moving Horizon Estimation performance

G. Sánchez*, M. Murillo, L. Giovanini

Research Institute for Signals, Systems and Computational Intelligence, sinc(i), FICH-UNL/CONICET, Ciudad Universitaria UNL, 4° piso FICH, S3000 Santa Fe, Argentina

ARTICLE INFO

Article history:
Received 12 May 2016
Received in revised form
28 October 2016
Accepted 17 February 2017

Keywords:
Moving Horizon Estimation
Arrival cost
Adaptive estimation
Constrained state estimation
Variable forgetting

ABSTRACT

Moving horizon estimation is an efficient technique to estimate states and parameters of constrained dynamical systems. It relies on the solution of a finite horizon optimization problem to compute the estimates, providing a natural framework to handle bounds and constraints on estimates, noises and parameters. However, the approximation of the arrival cost and its updating mechanism are an active research topic. The arrival cost is very important because it provides a mean to incorporate information from previous measurements to the current estimates and it is difficult to estimate its true value. In this work, we exploit the features of adaptive estimation methods to update the parameters of the arrival cost. We show that, having a better approximation of the arrival cost, the size of the optimization problem can be significantly reduced guaranteeing the stability and convergence of the estimates. These properties are illustrated through simulation studies.

1. Introduction

In control engineering, model–based control schemes assume that the states and parameters of the system are available for control law implementation. In practice, noisy measurements is the only information available from the system. Thus, the states and parameters have to be determined from these measurements using a dynamic model of the system. For linear systems, this problem has been solved and several methods based on different statistical measures have been developed [1,2].

The physical limits of the system can be modeled through bounds on its states and parameters. The omission of such information in the estimation algorithm may substantially hamper its performance [3]. Unfortunately, Kalman filter can not handle explicitly bounds on estimates (states and parameters) and ad hoc methods have been developed to handle constraints [4]. The different approaches to enforce constraints in Kalman filter include model reduction [5], estimate projection [6], gain projection [7], probability density function truncation [8] and system projection [9]. These methods yield to suboptimal solutions at the best and unrealistic estimates when the statistics of unknown variables (initial states, measurement and process noises, disturbances) are poorly chosen. On the other hand, moving horizon estimation (MHE) solves an optimization problem to find the system estimates, providing a theoretical framework for constrained estimation.

MHE solves at each sample a finite horizon state estimation problem to determine the states and parameters of the system. When new measurements become available the old ones are discarded from the estimation window, and the estimation problem is solved to determine the new estimates (see Fig. 1). The information of measurements which are not included in the estimation window is assimilated into the objective function through an extra term called *arrival cost*. It characterizes the statistical distribution of states at the beginning of the estimation window given the prior measurements information. In this way, it allows MHE to approximate the *full information* problem only considering a finite number of samples. A good approximation of the arrival cost allows to reduce the size of the estimation window and the size of the optimization problem, while it retains a good performance and robustness. The most accepted way of approximating the arrival cost is using a weighted 2-norm of the states at the beginning of the estimation window [10–13]. For linear systems Rao et al. [10] proposed to update the parameters of the arrival cost term (the weight matrix and initial states) using a Kalman filter or a Kalman smoother. The Gaussian distribution employed by these methods to model the conditional probabilities densities of initial states [10,14] results in inadequate approximation of the arrival cost when estimates are constrained, leading to poor and unrealistic estimates. This problem arises from the fact that the presence of bounds on estimates modifies the probability distribution of noises and estimates, forcing to zero the probability of some values and eliminating the independence between estimates and noises [15].

* Corresponding author.
E-mail addresses: gsanchez@sinc.unl.edu.ar (G. Sánchez), mmurillo@sinc.unl.edu.ar (M. Murillo), lgiovanini@sinc.unl.edu.ar (L. Giovanini).

http://dx.doi.org/10.1016/j.isatra.2017.02.012
0019-0578/

Please cite this article as: Sánchez G, et al. Adaptive arrival cost update for improving Moving Horizon Estimation performance. ISA Transactions (2017), http://dx.doi.org/10.1016/j.isatra.2017.02.012

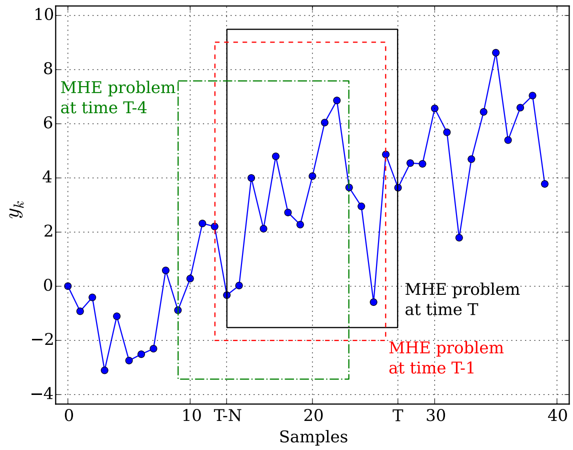

Fig. 1. MHE smoothing update.

To tackle these problems, Chu et al. [13] derived an iterative arrival cost update scheme that uses a quadratic approximation and information of active/inactive constraints from previous iteration. These ideas allow to build a quadratic approximation in the proximity of the optimal solution of the exact arrival cost. This update scheme is based on the hypothesis that the set of active constraints does not change after a particular time as the estimation horizon grows. This hypothesis works well when the estimation window is large, giving good results. However, if the hypothesis is not verified, i.e. some constraints become inactive after the state smoothing, the estimator may diverge. Furthermore, this updating mechanism overweights past data (by retaining active constraints) de-emphasizing the effect of information available in the new data on estimates. This fact can also cause divergence of the estimator if estimates are strongly correlated in time.

Finally, in a recent work Al-Matouq and Vincent [16] developed a MHE algorithm based on multiple estimation windows. The algorithm takes advantage of constraint inactivity to reduce the size of the optimization problem while it retains the stability and performance properties of full–information estimator. These properties are preserved by approximating the arrival cost with an unconstrained estimator that reformulates at each sample the objective function in the regions of constraints inactivity, allowing efficient long estimation windows.

In this work the weighting matrix is updated using adaptive estimation algorithms in combination with the MHE filter solution in the previous sample, while the initial states are the MHE filter solution in the previous sample. It should be noted that approximating the arrival in this way is consistent with constraints and bound on estimates and noises, while at the same time it introduces a feedback mechanism between measured data and estimates, improving the overall performance of the estimator. In this way, the weighting matrix is computed in a closed-loop fashion rather than in an open-loop way, as it is done in standard estimation techniques. The efficiency of the proposed approach is evaluated by conducting simulations studies on a benchmark problem available in the literature. The main contributions of this paper are to show that *i)* it is important to take into account bounds on estimates when the arrival cost is updated, *ii)* this idea can be implemented using adaptive estimation techniques to construct an approximation of the arrival cost, *iii)* the estimation resulting from a MHE with the proposed updating scheme is stable, and *iv)* the proposed updating scheme allows to shorten the estimation window without sacrificing performance, reducing the size of the optimization problem. The paper is organized as follows: In Section 2 the MHE problem is presented. In Section 3 two different ways of updating the arrival cost are analysed. Simulation results are discussed in Section 5 and the conclusions are outlined in Section 6.

Notation: In the sequel $x \in \mathbb{R}^n$ is a column vector and its transpose is denoted as x^T, $\mathbf{x}_k$ denotes a sequence of vectors over a given index up to k, for example $\mathbf{x}_k = \{x_j: j = 0, 1, \ldots, k\}$, $A \in \mathbb{R}^{n\times m}$ is a $n \times m$ matrix and its transpose is denoted by A^T. If $A \in \mathbb{R}^{n\times n}$ its inverse is denoted by A^{-1}. Given a symmetric real matrix $P \in \mathbb{R}^{n\times n}$, it is said to be positive definite if for all $z \in \mathbb{R}^n$, $z^T P z > 0$. In the following we will use the short notation $P > 0$ to denote that matrix P is positive definite. By $\|\cdot\|$ we denote the Euclidean vector or induced matrix norm. For $x \in \mathbb{R}^n$ and $P > 0 \in \mathbb{R}^{n\times n}$, we let $\| x \|_P = \sqrt{x^T P x}$.

2. Problem formulation

Consider the linear time invariant discrete system

$$x_{k+1} = Ax_k + Gw_k,$$
$$y_k = Cx_k + v_k, \tag{1}$$

where $x_k \in \mathcal{X} \subseteq \mathbb{R}^{n_x}$ is the state vector, $w_k \in \mathcal{W} \subseteq \mathbb{R}^{n_w}$ is the state noise vector, $y_k \in \mathbb{R}^{n_y}$ is the measurement vector, $v_k \in \mathcal{V} \subseteq \mathbb{R}^{n_y}$ is the measurement noise vector and $k \in \mathbb{I}$. In the following, $\hat{w}_k$ and $\hat{v}_k$ are considered zero mean stationary stochastic disturbances with finite moments and the restriction sets $\mathcal{X}$, $\mathcal{W}$ and $\mathcal{V}$ are considered closed with $\mathbf{0} \in \mathcal{X}$, $\mathbf{0} \in \mathcal{W}$ and $\mathbf{0} \in \mathcal{V}$.

The full information estimator[1] (*FIE*) problem uses a sequence of measurements y_k to find the state estimates $\hat{x}_k$ that solve the following optimization problem

$$\min_{\hat{x}_{0|k}, \hat{\mathbf{w}}_k} \Phi_k = \left\| \hat{x}_{0|k} - \bar{x}_0 \right\|^2_{P_0^{-1}} + \sum_{j=0}^{k} \left\| \hat{w}_{j|k} \right\|^2_{Q^{-1}} + \left\| \hat{v}_{j|k} \right\|^2_{R^{-1}}$$
$$\text{s. t.} \begin{cases} \hat{x}_{j+1|k} = A\hat{x}_{j|k} + \hat{w}_{j|k}, \\ y_j = C\hat{x}_{j|k} + \hat{v}_{j|k}, \\ \hat{x}_{j|k} \in \mathcal{X}, \hat{w}_{j|k} \in \mathcal{W}, \hat{v}_{j|k} \in \mathcal{V}, \end{cases} \tag{2}$$

where Q^{-1} and R^{-1} are symmetric positive definite matrices that penalize the estimated noise vectors $\hat{w}_{j|k}$ and the output prediction $\hat{v}_{j|k}$ error, respectively, and the pair $(\bar{x}_0, P_0^{-1})$ summarizes the prior information at time $k=0$ where $\bar{x}_0$ is the current knowledge of the initial estimate and P_0^{-1} is a symmetric positive definite weighting matrix. The solution of problem (2) yields smoothed estimates $\{\hat{x}_{j|k}: j = 0, 1, \ldots, k-1\}$, the filtered estimate $\hat{x}_{k|k}$ and sequence of estimated noise vectors, denoted as $\hat{\mathbf{w}}_k$ and $\hat{\mathbf{v}}_k$.

Since the full information estimator uses all measurements, as new ones become available the problem size grows with time making it intractable. MHE overcomes this problem by only considering a fixed amount of data and dynamic updates by sliding a window with time. The past measurements are taken into account through a penalty cost term Z_{k-N} called "*arrival cost*" [10]. Then, the MHE problem consists in finding the states, state and measurement noises that solve the following optimization problem

[1] Other authors use the term batch least squares