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Research article

A uniform LMI formulation for tuning PID, multi-term fractional-order PID, and Tilt-Integral-Derivative (TID) for integer and fractional-order processes

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ABSTRACT

In this paper first the Multi-term Fractional-Order PID (MFOPID) whose transfer function is equal to $\sum_{j=1}^{N} k_j s^{\alpha_j}$, where k_j and α_j are unknown and known real parameters respectively, is introduced. Without any loss of generality, a special form of MFOPID with transfer function $k_p + k_i/s + k_{d1}s + k_{d2}s^{\mu}$ where k_p, k_i, k_{d1}, and k_{d2} are unknown real and μ is a known positive real parameter, is considered. Similar to PID and TID, MFOPID is also linear in its parameters which makes it possible to study all of them in a same framework. Tuning the parameters of PID, TID, and MFOPID based on loop shaping using Linear Matrix Inequalities (LMIs) is discussed. For this purpose separate LMIs for closed-loop stability (of sufficient type) and adjusting different aspects of the open-loop frequency response are developed. The proposed LMIs for stability are obtained based on the Nyquist stability theorem and can be applied to both integer and fractional-order (not necessarily commensurate) processes which are either stable or have one unstable pole. Numerical simulations show that the performance of the four-variable MFOPID can compete the trivial five-variable FOPID and often excels PID and TID.

1. Introduction

Fractional-order PID (FOPID) was introduced in 1999 by Podlubny as a generalization of trivial PID controllers [1]. The transfer function of the ideal FOPID, also known as $PI^{\lambda}D^{\mu}$, is defined as [1]:

$$C_{FOPID}(s) = k_p + \frac{k_i}{s^{\lambda}} + k_d s^{\mu}, \tag{1}$$

where $k_p, k_i, k_d \in \mathbb{R}$ and $\lambda, \mu \in \mathbb{R}^+$ are unknown parameters of controller to be tuned. See, for example, [2] for the possible time-domain interpretations of the fractional powers of s in (1) and their properties. Considering the fact that FOPID has five parameters to tune, two more than the classical PID, it is expected that it leads to a higher performance compared to PID especially in dealing with problems with complicated control objectives [3]. Various successful applications of FOPID controllers have been reported in the literature. Some examples are motion control [4], unmanned aerial vehicle [5], path tracking control of tractors [6], and control of a solar furnace [7]. FOPID controllers can be realized using either analogue [8,9] or digital techniques [10,11].

Another fractional-order controller which is closely related to FOPID and discussed in this paper is the Tilt-Integral-Derivative (TID). The transfer function of TID is defined as [12]:

$$C_{TID}(s) = \frac{k_t}{s^{1/n}} + \frac{k_i}{s} + k_d s, \tag{2}$$

where again k_t, k_i, and k_d are unknown real parameters to be calculated, and n is an unknown positive integer often considered equal to 2 or 3 before tuning other parameters [12]. Some properties of TID have been studied in [13]. A method for tuning its parameters is also presented therein.

So far, a wide variety of techniques have been proposed for tuning the parameters of FOPID. For example, tuning the parameters based on open-loop shaping [14,3], Bode's ideal transfer function [15,16], minimization of performance indices like ISE and IAE [17,18], optimization of load disturbance subject to a constraint on the maximum sensitivity (M_s) [19], simultaneous adjustment of phase and gain margin to the desired values [20], and fractional-order root-locus method [21] can be found in the literature. Some of these methods like those presented in [17] and [18] lead to explicit tuning rules which can be applied to first-order plus time delay (FOPTD) or integrator plus dead time (IPDT) or unstable first-order plus dead-time (UFOPDT) processes. It is worth mentioning that almost all of the methods developed so far for tuning FOPID controllers are based on solving a kind of optimization problem. Some researchers have used meta-heuristic optimization algorithms for solving such problems; see for example [22–24].

During the past decades many control problems have been successfully formulated as convex optimization problems

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involving Linear Matrix Inequalities (LMIs) [25]. Recently, some researchers have applied LMIs for stability analysis and controller synthesis for fractional-order systems (FOS). For example, LMI approach for stability analysis of a system governed by the state-space equation $D^{\nu}x(t) = Ax(t)$ where D^{ν} is a fractional-order derivative operator, $A \in \mathbb{R}^{n\times n}$, $x(t) \in \mathbb{R}^{n}$ and $0 < \nu < 2$ is presented in [26]. LMI-based sufficient conditions for robust stability and stabilization of linear time-invariant FOS are presented in [27]. In the field of synthesis of control laws, pseudo-state feedback stabilization of commensurate FOS [28] and H_∞ output feedback control of commensurate FOS [29], both using LMIs, are reported in the literature. LMIs for calculating the bounds on the norms of FOS are presented in [30].

The aim of this paper is to propose a method for tuning a kind of FOPID using LMIs. For this purpose first a new fractional-order controller which is defined by adding a fractional differentiator of fixed order to the classical PID, is introduced. Then LMIs for shaping the open-loop frequency response when the proposed controller is applied are developed. The proposed approach makes it possible to use advantages of convex optimization and fractional-order operators to solve a control problem.

The rest of this paper is organized as the following. The main results including the proposed structure for fractional-order controller and the corresponding LMIs for open-loop shaping are presented in Section 2. Four illustrative examples are presented in Section 3, and Section 4 concludes the paper.

2. Multi-term FOPID: definition and tuning

In this section first the multi-term FOPID (MFOPID) is introduced. Then separate LMIs for closed-loop stability (of sufficient type), adjustment of phase margin (PM), adjustment of only the open-loop phase, and adjustment of only the open-loop gain are presented. In practice, the user can apply the LMIs for stability in combination with any of the other LMIs to design the controller by loop shaping. At the end of this section another approach for controller design which is also based on LMIs and applicable to higher order and time-delayed processes with one unstable pole is presented.

2.1. The proposed multi-term FOPID

Before introducing the MFOPID controller first note that unlike FOPID, PID and TID (assuming a fixed value for n in (2)) are linear in their parameters. More precisely, considering the vector of unknown parameters as

$$X = \left[k_p\, k_i\, k_d\right]^{\mathrm{T}}, \tag{3}$$

the classical ideal PID can be expressed as

$$C_{PID}(s) = k_p + \frac{k_i}{s} + k_d s = \left[1\ s^{-1} s\right]X = W_{PID}(s)X, \tag{4}$$

which is linear in X. Similarly, assuming

$$X = \left[k_t\, k_i\, k_d\right]^{\mathrm{T}}, \tag{5}$$

the TID defined in (2) can be expressed as

$$C_{TID}(s) = \left[s^{-1/n} s^{-1} s\right]X = W_{TID}(s)X, \tag{6}$$

which again assuming a certain value for n is linear in X. But the FOPID given in (1) cannot be expressed in the same way unless the values of λ and μ are assumed to be fixed in advance. The possibility of writing a controller with transfer function $C(s)$ in the form of $C(s) = W(s)X$, where $W(s)$ and X are the known *weights* and the unknown *parameters* vectors, respectively, is an advantage since it makes it possible to calculate X through LMIs and convex optimization algorithms as it will be discussed in the following. The above discussion motivates us to present a new definition for FOPID controller which is firstly linear in the vector of tuning parameters, and secondly, has more tuning parameters compared to PID and can mimic the performance of traditional FOPID defined in (1).

According to the above discussion the transfer function of MFOPID is defined as

$$C_{MFOPID}(s) \triangleq \sum_{j=1}^{N} k_j s^{\alpha_j} = \left[s^{\alpha_1} s^{\alpha_2} \ldots s^{\alpha_N}\right]X = W_{MFOPID}(s)X, \tag{7}$$

where $\alpha_1, \ldots, \alpha_N$ are known real constants, and

$$X = \left[k_1 \ldots k_N\right]^{\mathrm{T}}, \tag{8}$$

is the vector of real unknown parameters, and

$$W_{MFOPID}(s) = \left[s^{\alpha_1} s^{\alpha_2} \ldots s^{\alpha_N}\right], \tag{9}$$

is the weights vector which is known at each frequency.

Without any loss of generality, a special form of (7) as given in (10) is used in the numerical examples of this paper:

$$C_{MFOPID}(s) = k_p + \frac{k_i}{s} + k_{d1}s + k_{d2}s^{\mu}, \tag{10}$$

where $0 < \mu < 2$ is a pre-determined positive real constant and the vector of unknown parameters is $X = \left[k_p\, k_i\, k_{d1}\, k_{d2}\right]^{\mathrm{T}}$. The controller in (10) can be though of as a PID accompanied with an extra fractional-order derivative operator. The reason for being interested in this controller is that the traditional integrator $1/s$ is sufficient for many applications while there is often a need for an extra phase lead which can be achieved by the term $k_{d2}s^{\mu}$ in (10). Note that in practice the order of integrator, λ, in (1) is often considered larger than or equal to unity since application of λ's smaller than unity leads to very slow convergence of the closed-loop step response to its final value, which is not desired. In the rest of this paper the transfer function of controller is considered as $C(s) = W(s)X$ for some $W(s)$ and X.

2.2. LMIs for closed-loop stability (stable process)

Consider the feedback system shown in Fig. 1 where here $P(s)$ is assumed to be stable. In this case it is concluded from the Nyquist stability theorem that the closed-loop system is also stable if and only if the Nyquist plot of $L(s) = C(s)P(s) = W(s)XP(s)$ does not encircle -1. Clearly, infinity many different Nyquist plots can be drawn which satisfy this condition. Fig. 2 shows a common approach to achieve closed-loop stability in this case where the Nyquist plot lies in the lower (upper) half-plane at all frequencies smaller (larger) than the phase crossover frequency, ω_{pc}, and at $\omega = \omega_{pc}$ we have $\mathrm{Re}\{L(j\omega_{pc})\} > -1$ and $\mathrm{Im}\{L(j\omega_{pc})\} = 0$. According to this figure the *sufficient* condition for closed-loop stability is the simultaneous satisfaction of (11)–(13):

$$\mathrm{Im}\{L(j\omega)\} < 0,\ 0 \le \omega < \omega_{pc}, \tag{11}$$

$$\mathrm{Re}\{L(j\omega_{pc})\} > -1 \quad \wedge \quad \mathrm{Im}\{L(j\omega_{pc})\} = 0, \tag{12}$$

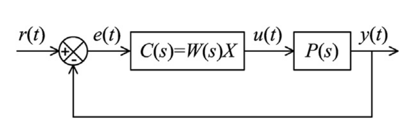

Fig. 1. The closed-loop system under consideration.

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