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Research article

Finite-time stability and stabilization for stochastic markov jump systems with mode-dependent time delays ☆

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ABSTRACT

This paper is concerned with the problems of finite-time stability and stabilization for stochastic Markov systems with mode-dependent time-delays. In order to reduce conservatism, a mode-dependent approach is utilized. Based on the derived stability conditions, state-feedback controller and observer-based controller are designed, respectively. A new N-mode algorithm is given to obtain the maximum value of time-delay. Finally, an example is used to show the merit of the proposed results.

1. Introduction

Markov jump systems have been widely studied due to their widely practical applications in economic systems [1], power systems [2] and so on. A great deal of results on such class of systems have been obtained, e.g., [3–9]. And also, time-delay is often encountered in practical process, such as communication systems [10] and networked control systems [11,12]. Considering the above practical phenomena, a class of more general model of Markov jump systems with time-delays have been paid more attention gradually. Many interesting results have been obtained for this kind of systems. For example, [13] gave some sufficient conditions for stochastic stability of Markov jump systems with time-delay and partially known transition probabilities. The stability of delayed recurrent neural networks with impulse control and Markovian jump parameters was investigated in [14]. The literature [15] addressed $\mathcal{H}_\infty$ performance analysis problem for singular Markov jump delayed systems with polyhedral parameter uncertainties and [16] investigated delayed state feedback stabilization of uncertain Markovian jump linear systems with random Markov delays.

Recently, finite-time stability has also received increasing attention and been found some applications in practical process, such as, avoiding saturation or the excitation of nonlinear dynamics during the transient [17]. The concept of finite-time stability was first introduced in the 1950s and played an important role in addressing transient performances of control systems. Roughly speaking, a system is said to be finite-time stable if for a given time-interval $[0, T]$, its states can not exceed a specified bound in the time interval [18]. Many interesting results have been obtained for this type of stability. For example, [19] investigated the problems of finite-time stability and stabilization for Itô stochastic system and [20] addressed finite-time stochastic contractive bounded-ness of Markovian jump systems. The literature [21] studied observer-based state feedback finite-time control for nonlinear jump systems with time-delay. and [22] investigated finite-time $\mathcal{H}_\infty$ control for Markovian jump systems with mode-dependent time-varying delays.

Although several interesting results on the problems of finite-time stability and stabilization have been reported, it has not yet been fully investigated for Markovian jump systems with mode-dependent time-delays. In these works, a key method is to construct the following inequality [20–22]

$$\frac{d[x'(t)P_i x(t)]}{dt} < \alpha[x'(t)P_i x(t)], \tag{1}$$

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where only P_i is mode-dependent, while α is common to all modes. This implies α should satisfy inequality (1) for all modes, which probably results in some conservative conditions. If α is also mode-dependent, some less conservative conditions may be obtained. This is because mode-dependent α, that is α_i, is more flexible than the common one. Here, we call this approach mode-dependent approach(MDA). On the other hand, in these works, there is nearly no literatures to investigate the problems of finite-time stability and stabilization on Itô stochastic Markovian jump systems with mode-dependent time-delays.

Motivated by aforementioned discussions, we use a mode-dependent approach to study the problems of finite-time stability and stabilization for Itô stochastic Markov jump systems with mode-dependent time-delays. The system model addressed is more complex than those in existing literatures, which results in the difficulty of stability analysis and controller design. By utilizing stochastic analysis technology, a stability condition and some stabilizing conditions are derived. The main contributions of this paper are as follows. 1) The definition of finite-time stability is extended to the model of Itô stochastic Markov jump systems with mode-dependent time-delays. 2) A stability condition and two new sufficient conditions of the existence for state feedback and observer-based controllers are given by a mode-dependent approach, which are of less conservativeness. 3) A new N-mode algorithm is provided for obtaining the maximum value of time-delay.

The structure of this paper is organized as follows. In Section 2, we give some preliminaries and the definitions of finite-time stability and stabilization. In Section 3, a finite-time stability condition is given. Section 4 provides some sufficient conditions for the existence of a state feedback and a observer-based controller. Section 5 gives an algorithm to obtain the maximum value of time-delay. An example is employed to illustrate the results in Section 6.

Notations: X' stands for transpose of a matrix X. The notation $Q>0$ means that Q is positive definite. $\lambda_{\max}(X)(\lambda_{\min}(X))$ represents the maximum (minimum) eigenvalue of a matrix X. $I_{n\times n}$ stands for $n\times n$ identity matrix. $\mathbb{E}[X]$ denotes the expectation of X. We use the asterisk $*$ in a matrix to represent the term which is induced by symmetry. The "wrt" is an abbreviation of "with respect to".

2. Definitions and preliminaries

Let $w(t)$ be a scalar Brownian motion defined on the probability space $(\Omega, \mathcal{F}, P)$. Let r_t be a right-continuous Markov chain with the state space $\Gamma=\{1,2,\ldots,N\}$ and the transition rate matrix $\pi=[\pi_{ij}]_{N\times N}$. We assume that r_t is independent of $w(t)$ and has the following transition probability

$$P\{r_{t+\triangle t}=j|r_t=i\}=\begin{cases}\pi_{ij}\triangle t+o(\triangle t), & i\neq j,\\ 1+\pi_{ij}\triangle t+o(\triangle t), & i=j,\end{cases}$$

where $\triangle t>0$, π_{ij} is the stationary transition rate from mode i to mode j, which satisfies $\pi_{ij}>0,\ i\neq j$ and $\pi_{ii}=-\sum_{j=1,\,i\neq j}^{N}\pi_{ij}$. $\mathcal{F}_t$ stands for the smallest σ-algebra generated by $w(s)$, $r(s)$, $0\le s\le t$, i.e., $\mathcal{F}_t=\sigma\{w(s), r(s)|0\le s\le t\}$.

Consider the following Itô stochastic Markov system with mode-dependent time-delays

$$\begin{cases}dx(t)=[A(r_t)x(t)+A_1(r_t)x(t-h(r_t))]dt\\ \qquad +[\overline{A}(r_t)x(t)+\overline{A}_1(r_t)x(t-h(r_t))]dw(t)\\ y(t)=C(r_t)x(t),\\ x(t)=\varphi(t),\ \ r(t)=r(0),\ \ \forall\, t\in[-h,0],\end{cases} \tag{2}$$

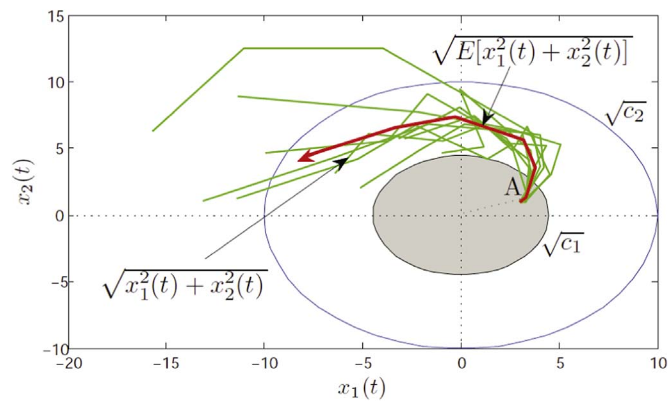

Fig. 1. Illustration of FTS.

where $x(t)\in\mathcal{R}^n$, $u(t)\in\mathcal{R}^m$, and $y(t)\in\mathcal{R}^p$ are state, control input and measurement output, respectively. For $r_t=i$, $A(r_t)$, $A_1(r_t)$, $\overline{A}(r_t)$, $\overline{A}_1(r_t)$, $B(r_t)$, $\overline{B}(r_t)$ are constant matrices of compatible dimensions, denoted by A_i, A_{1i} $\overline{A}_i$, $\overline{A}_{1i}$, B_i, $\overline{B}_i$ for simplicity. $\varphi(t)$ is a initial function and the scalar $h>0$ denotes an unknown system delay.

Next, the definition of finite-time stochastic stability for Itô stochastic Markov system with mode-dependent time-delays is introduced.

Definition 1. Given positive real scalars c_1, c_2, T with $0<c_1<c_2$, and a positive definite matrix R, the system (2) is said to be finite-time stochastically stable(FTSS) wrt (c_1, c_2, T, R), if

$$\sup_{-h\le t_0\le 0}\{x'(t_0)Rx(t_0)\}\le c_1\Rightarrow\mathbb{E}[x'(t)Rx(t)]<c_2,\quad t\in[0,\ T].$$

Remark 1. Definition 1 can be described as following: if a given bound on the initial condition, a fixed time interval and $R=I$, the expected value of state trajectories of system (2) is required to remain in a certain domain during this time interval. A two-dimensional case of Definition 1 is illustrated by Fig. 1. A point **A** lies in the shaped area. The trajectory starting from **A** can not escape the domain of radius of $\sqrt{c_2}$ during the time interval $[0,\ T]$.

Remark 2. Finite-time stochastic stability requires the expected values of the states not to exceed a given bound in finite-time interval, which is different from mean square asymptotic stability [23]. A system that is mean square asymptotically stable may be not FTSS, if the expected values of its states exceed a given upper bound, and vice versa.

Next, consider the Itô stochastic Markov controlled system with mode-dependent time-delays

$$\begin{cases}dx(t)=[A(r_t)x(t)+A_1(r_t)x(t-h(r_t))+B(r_t)u(t)]dt\\ \qquad +[\overline{A}(r_t)x(t)+\overline{A}_1(r_t)x(t-h(r_t))+\overline{B}(r_t)u(t)]dw(t),\\ y(t)=C(r_t)x(t),\\ x(t)=\varphi(t),\ \ r(t)=r(0),\ \ \forall\, t\in[-h,0].\end{cases} \tag{3}$$

On the basis of Definition 1, the definition of finite-time stochastic stabilization can be given as follows.

Definition 2. System (3) is said to be finite-time stochastically stabilizable if there exists a feedback control law $u^*(t)$, such that

$$dx(t)=[A(r_t)x(t)+A_1(r_t)x(t-h(r_t))+B(r_t)u^*(t)]dt$$
$$+[\overline{A}(r_t)x(t)+\overline{A}_1(r_t)\,x(t-h(r_t))+\overline{B}(r_t)u^*(t)]dw(t) \tag{4}$$

is finite-time stochastically stable wrt (c_1, c_2, T, R).

The following lemma will be used in the next section.

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