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Research article

Fractional order $PI^{\lambda}D^{\mu}$ controller design for satisfying time and frequency domain specifications simultaneously

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ABSTRACT

In order to achieve a desired control performance characterized by satisfying specifications in both frequency-domain and time-domain simultaneously, an optimal fractional order proportional integral derivative ($PI^{\lambda}D^{\mu}$) controller design strategy is proposed based on analytical calculation and Differential Evolution algorithm for a permanent magnet synchronous motor (PMSM) servo system in this paper. In this controller design, the frequency-domain specifications can guarantee the system stability with both gain margin and phase margin, and also the system robustness to loop gain variations. The time-domain specifications can ensure the desired step response performance with rapid rising curve, constrained overshoot, and proper power consuming. Compared with the PI^{λ} controller and the traditional PID controller, $PI^{\lambda}D^{\mu}$ controller can get obvious benefits from two more degrees of freedom of the fractional orders λ and μ on satisfying multiple constraints simultaneously and achieving better servo tracking performance for the PMSM servo system. PMSM speed tracking simulations and experiments are demonstrated to show the significant advantages of using the proposed optimal $PI^{\lambda}D^{\mu}$ controller over the optimal fractional order PI^{λ} controller and traditional integer order PID controller.

1. Introduction

In recent years, fractional calculus has been widely used in system modeling [1–4] and control area [5–9]. The characteristics of real-world systems can be described more precisely using fractional order mathematical models [10,11]. Meanwhile, proportional integral derivative (PID) control has been the most widely used and developed control method in industrial control area [12–15]. Fractional order proportional integral derivative ($PI^{\lambda}D^{\mu}$) controller has the potential to achieve better control performance over the traditional PID controller because the differential order and integral order are introduced as adjustable controller parameters, which increase the flexibility of the controller [16,7–9,17–20]. But the tuning of $PI^{\lambda}D^{\mu}$ controller is more complicated over the traditional PID controller because two extra parameters are added [16,21–25]. Especially, how to design an optimized $PI^{\lambda}D^{\mu}$ controller to achieve desired performance specified in both frequency-domain and time-domain is deserved to be investigated.

In present, the tuning methods for the fractional order PI^{λ}/D^{μ} controller mainly contain the frequency-domain design methods [26] and other time-domain optimization methods [27–29]. The frequency-domain method is often applied to design the fractional order PI^{λ} or PD^{μ} controllers [21,26]. As presented in [26], based on the given gain crossover frequency and phase margin, the controller parameters are calculated according to the gain robustness specification. The obtained control system can achieve the robustness to gain variations. However, this frequency-domain method may not be directly applied to design the fractional order $PI^{\lambda}D^{\mu}$ controller, because two extra parameters are introduced. Meanwhile, the system gain margin is an important stability index in real industrial control applications, but it is always ignored in the frequency-domain method. The time-domain optimization methods search for the optimal controller parameters by optimizing an objective function [27,28]. The obtained control system can achieve the optimal time-domain dynamic performance. But the system stability with gain and phase margin, and the robustness performance specified in frequency-domain may not be able to be guaranteed simultaneously.

In order to obtain a controller to achieve good dynamic performance under the condition that both the requirements in time-domain and frequency-domain are satisfied, a fractional order $PI^{\lambda}D^{\mu}$ controller design strategy is proposed in this paper and a fractional order $PI^{\lambda}D^{\mu}$ controller is designed for a permanent magnet synchronous motor (PMSM) servo system, based on

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analytical calculation and Differential Evolution (DE) algorithm [30]. In frequency-domain, taking the loop-gain robustness specification as a constraint condition, the gain margin and phase margin specifications as the boundary conditions, in time-domain, taking the integral of time and absolute error (ITAE) [31] as the objective function, the step response overshoot upper-limit as the dynamic threshold, the power consumption upper-limit as the energy threshold, an optimal $PI^{\lambda}D^{\mu}$ controller can be obtained by using the DE algorithm [30]. Under this controller design strategy, the frequency-domain specifications can guarantee the control system stability with not only phase margin but also gain margin; the system robustness to loop-gain variations can also be satisfied from a flat-phase specification in frequency-domain; the time-domain specifications can ensure the desired step response performance with rapid rising curve, constrained overshoot, and proper power consuming.

Using this proposed controller design method, the fractional order $PI^{\lambda}D^{\mu}$ controller can obtain obvious benefits from two more degrees of freedom of the fractional orders λ and μ. This designed $PI^{\lambda}D^{\mu}$ is able to satisfy multiple constraints in both frequency-domain and time-domain simultaneously. Especially, it can achieve better servo tracking performance over the traditional PID controller for the PMSM servo system with a fractional order model which can describe the real PMSM servo system more precisely over the integer order model [32]. PMSM speed-tracking simulations and experiments are demonstrated to show the advantages of the proposed tuning method over the methods proposed in [29]. Besides, the significant advantages of using the proposed $PI^{\lambda}D^{\mu}$ controller over the fractional order PI^{λ} controller and traditional integer order PID controller are also demonstrated.

The rest of this paper is arranged as follows: the model of the PMSM servo control system is discussed in Section 2; the fractional order $PI^{\lambda}D^{\mu}$ controller design method is proposed in Section 3; PMSM speed control simulations are presented in Section 4. The obtained $PI^{\lambda}D^{\mu}$ controller is compared with those obtained using the time-domain and frequency-domain tuning methods proposed in [29]. The dynamic performance of the obtained $PI^{\lambda}D^{\mu}$ controller is also studied by comparison with the PI^{λ} controller and traditional PID controller; real PMSM speed control experiments are presented in Section 5; the conclusion is given in Section 6.

2. PMSM speed control system

According to our previous work [32], a fractional order model is able to describe the real PMSM servo system more precisely over the integer order model. Therefore, the fractional order model is applied for the PMSM servo system controller design in this paper. The block diagram of the fractional order model of the PMSM speed control system is shown in Fig. 1, where n_r is the reference speed, n is the actual speed, $C_v(s)$ is the speed controller, i_{qr} is the speed controller output, i_q is the q-axix current, $C_i(s)$ is the current controller, K_0 is the voltage conversion factor, K_1 is the current conversion factor, T_i is the current filter time constant, K_2 is the speed conversion factor, R is the resistor, L is the inductor, C_m is the torque constant, GD^2 is the flywheel inertia, C_e is the induced voltage constant, $\vartheta \in (0, 2)$ and $\zeta \in (0, 2)$ are the fractional orders in the model.

In order to obtain the plant model of the speed control system, the current loop is properly simplified. Since the changing of the current is much faster than that of the motor speed, the induced voltage E can be considered unchanging when studying the variations of the current. Therefore, the influence of the induced voltage E can be eliminated [33] and the current loop is simplified as shown in Fig. 2.

A proportional-integral (PI) controller is chosen to be the current controller, whose integral time constant is set to be T_i, as described by (1),

$$C_i(s) = K_{pi}\left(1 + \frac{1}{T_i s}\right). \tag{1}$$

Then the current loop model is simplified as shown in Fig. 3, where

$$\kappa = \frac{K_{pi}K_0}{T_i L}. \tag{2}$$

Therefore, the closed-loop transfer function of the current loop is described by (3),

$$G_i(s) = \frac{\kappa}{s^{\vartheta+1} + \frac{R}{L}s + K_1\kappa}. \tag{3}$$

The speed control loop can be converted into an unit feedback control system, as shown in Fig. 4.

The plant model of this speed control system can be generalized as the following form,

$$G(s) = \frac{d}{s^{\alpha} + as^{\beta} + bs^{\gamma} + c}, \tag{4}$$

where $\alpha = \xi + \vartheta + 1$, $\beta = \xi + 1$, $\gamma = \xi$, $a = \frac{R}{L}$, $b = K_1\kappa$, $c=0$, $d = \frac{375C_mK_2\kappa}{GD^2}$.

The fractional order $PI^{\lambda}D^{\mu}$ controller is designed based on this PMSM speed control model described by (4) in this paper.

3. Fractional order $PI^{\lambda}D^{\mu}$ optimal design method

The fractional order $PI^{\lambda}D^{\mu}$ controller is described by (5),

$$C(s) = K_p\left(1 + \frac{K_i}{s^{\lambda}} + K_d s^{\mu}\right), \tag{5}$$

where, K_p, K_i and K_d are proportional, integral and derivative gains, respectively; $\lambda \in (0, 2)$ and $\mu \in (0, 2)$ are the fractional orders.

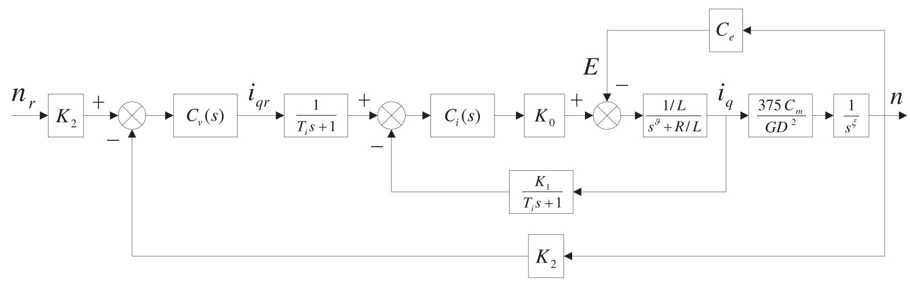

Fig. 1. Diagram of the feedback control system.

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