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An adaptation of adomian decomposition for numeric–analytic integration of strongly nonlinear and chaotic oscillators

S. Ghosh, A. Roy, D. Roy *

Department of Civil Engineering, Indian Institute of Science, Bangalore 560012, India

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Abstract

A novel form of an explicit numeric-analytic technique is developed for solving strongly nonlinear oscillators of engineering interest. The analytic part of this technique makes use of Adomian Decomposition Method (ADM), but unlike other analytical solutions it does not rely on the functional form of the solution over the whole domain of the independent variable. Instead it discretizes the domain and solves multiple IVPs recursively. ADM uses a rearranged Taylor series expansion about a function and finds a series of functions which add up to generate the required solution. The present method discretizes the axis of the independent variable and only collects lower powers of the chosen step size in series solution. Each function constituting the series solution is found analytically. It is next shown that the modified ADM can be used to obtain the analytical solution,in a piecewise form. For nonlinear oscillators such a piecewise solution is valid only within a chosen time step. An attempt has been made to address few issues like the order of local error and convergence of the method. Emphasis has been on the application of the present method to a number of well known oscillators. The method has the advantage of giving a functional form of the solution within each time interval thus one has access to finer details of the solution over the interval. This is not possible in purely numerical techniques like the Runge–Kutta method, which provides solution only at the two ends of a given time interval, provided that the interval is chosen small enough for convergence. It is shown that the present technique successfully overcomes many limitations of the conventional form of ADM. The present method has the versatility and advantages of numerical methods for being applied directly to highly nonlinear problems and also have the elegance and other benefits of analytical techniques.

Keywords: Adomian decomposition method; Nonlinear oscillators; Chaos

1. Introduction

In many practical situations a system of coupled, possibly damped, nonlinear ordinary differential equations model the dynamical behavior of mechanical systems. For example, these equations arise (following some discretization procedure) while studying the mechanical response of systems such as strings, beams, absorbers, plates, and so on. In general, exact solutions of such equations are unknown and thus numerical integration, perturbation techniques or geometrical methods (see [1–4] and references there in) have been applied to obtain their approximate solutions. However, in many of the analytical techniques, it becomes necessary to resort to linearization techniques or assumption of weak nonlinearity, except for a small class of low-dimensional problems which can be transformed to linear equations. This so-called weak nonlinearity or small parameter assumption greatly restricts applications of perturbation techniques known that an overwhelming majority of nonlinear problems have no small parameters at all. Therefore such analytic routes may not be able to treat strongly nonlinear problems. Recently there are few attempts to overcome this restriction of weak nonlinearity (see, for instance [5–7]), but they

* Corresponding author. Tel.: +91 80 22933129; fax: +91 80 23600404.
E-mail address: royd@civil.iisc.ernet.in (D. Roy).

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either use the small parameter assumption indirectly or lack the versatility and applicability to all kinds of problems. Numerical integration methods, even though more versatile than their analytical counterparts, sometimes respond too sensitively to the choice of time-step size to be reliable (see, for instance, Yamaguti and Ushiki [8]; Lorenz [9]). Moreover, numerical integration schemes always offer approximate solutions in a discretized form thereby making it difficult to obtain a continuous representation. A third route, based on numeric-analytical algorithms, has also been tried out for nonlinear deterministic and stochastic initial value problems (IVPs). The phase-space linearization (PSL) method, proposed by Roy [10,11] and Iyengar and Roy [12,13] and implicit local linearization method termed as the Locally Transversal Linearization (LTL) [4,14] are two such ways. The present objective is to follow them up with yet another numerical-analytic technique based on Adomian Decomposition technique. George Adomian introduced a method in [15] for solving nonlinear functional equations of various kinds (algebraic, differential, delay differential, etc.), and the method is known as Adomian Decomposition Method (ADM). This method and its application are well documented in [16–22]. The technique uses a decomposition of the nonlinear term into a series of functions. Each term of this series is a generalized polynomial called Adomian's polynomial. These polynomials generate an infinite set of functions whose sum determines the actual solution. This method has significant advantages: it is not restricted with weak nonlinearity assumption, and provides a rapidly convergent series [19]. A more detailed description about the mathematical background of this method is available in [22]. Another important work by Rach [20] is worthy of mention here. It makes a comparison between Picard's method and ADM, and concludes that these two methods are not the same, with Picard's method being applicable only if the vector field satisfies the Lipschitz condition. The concept of noise terms is introduced in [23] for a faster convergence of decomposed series solution. Noise terms have been defined as the identical terms with opposite signs that appear in the first two terms of the series solution. It is also shown that noise terms appear always for inhomogeneous equations. Though easy computations of series terms are closely related to global accuracy and applicability of the method, this aspect has never been explored in previous works. In Section 2 of this study, it is demonstrated that the finite series approximation is inaccurate. Unless the decomposed solution is valid over long time intervals, easier computations of series terms are not of importance. Indeed, in the context of integration of equations of motion of nonlinear oscillators, the global accuracy of the method needs to be addressed rigorously. To the authors' knowledge such issues have not been addressed, especially for simulations of strongly nonlinear and possibly chaotic oscillators using ADM.

It has been stated that the proof of convergence of Adomian series may be based on

(1) the fixed point theorem [19,22];
(2) the assumption that the nonlinear function is replaceable by an infinite series with a convergence radius equal to infinity [24].

Supplemented by the assumption that the convergence is fast. But for all practical purposes, this series converges only locally because it is based on a Taylor series expansion about an initial function which is often monotonically increasing. This observation will be substantiated in the present work through application of the conventional ADM to dynamical system. This lack of global convergence is overcome here by recursively applying ADM over successive time intervals, thereby solving a sequence of IVPs, each valid over a given time interval. However, it is found that this discretization technique fails to provide accurate solutions when there are forcing terms and in several other cases where the system's nonlinearity is of a nonpolynomial form. The reason behind is that it is too complicated to compute series terms other than first few in such cases using the conventional ADM. So, the forcing function is also expanded in a Taylor series and appropriately distributed in all the series terms. Finally, terms with higher powers of increments of the independent variable are removed. This provides a systematic algorithm for the analytical computation of series terms. Indeed the removal of higher power helps considerably in simplifying the analytical derivation of series terms. These techniques will be discussed in Section 3. Computation of local error and its global propagation is investigated theoretically as well as with help of examples and compared with existing numerical techniques. Special attention is given to applicability of this method to oscillators responding in the chaotic regime. The present method demands the usage of extensive symbolic computation in obtaining the series terms. But this method has certain advantages as follows,

(1) Conventional ADM gives only locally convergent results (see Section 2.1). It is shown that the present method successfully overcomes such limitations and has applicability to a large class of nonlinear dynamical systems of engineering interest.
(2) Like other numerical techniques this method is also versatile and can solve strongly nonlinear and chaotic systems – something not possible through purely analytical techniques.
(3) The present method can provide a piecewise functional form of the solution within each discrete interval. The approximated solution may thus be made as smooth as desired, which is not possible with any purely numerical technique like the Runge–Kutta method as the latter merely provides a discretized approximation of the true solution.
(4) The order of accuracy is quite high and may be further increased by simply generating more series terms without decreasing the time-step size.