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Dissipativity analysis for discrete singular systems with time-varying delay ☆

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ABSTRACT

In this paper, the issue of dissipativity analysis for discrete singular systems with time-varying delay is investigated. By using a recently developed inequality, which is less conservative than the Jensen inequality, and the improved reciprocally convex combination approach, sufficient criteria are established to guarantee the admissibility and dissipativity of the considered system. Moreover, H_∞ performance characterization and passivity analysis are carried out. Numerical examples are presented to illustrate the effectiveness of the proposed method.

1. Introduction

The theory of dissipative systems has drawn many researchers' attention since it was first proposed in 1972 by Willems [1], due to its very wide applications in many fields such as system theory, circuit design, network synthesis and control theory [2,3]. Dissipativity theory is built on an input–output energy-related consideration to a framework for the design and analysis of control systems [4,5]. It is a generalization of H_∞ and passivity performance and shows a strong connection among physics, systems theory and control engineering [6]. Therefore, dissipativity theory has much good performance on both linear and non-linear systems [7].

In recent years, the interest in the research of singular systems is growing since such systems have very extensive applications in electrical network analysis, economic systems, and large-scale systems [8–10]. Also referred to as generalized state-space systems, semi-state systems or descriptor systems, singular systems are more complicated to study [11], because not only the stability of the system should be taken into account, but also regularity and absence of impulses (for continuous singular systems) or causality (for discrete singular systems), while the latter two issues do not appear in standard state-space systems [12]. On the other hand, time delays widely exsit in many practical systems and are the main causes of instability and poor performance of dynamic systems [13,14]. Hence, many studies towards time-delay systems have been carried out [15,16].

One of the main methods to solve the control problem of discrete time-delay singular systems is the robust H_∞ control, and many results on robust H_∞ control of singular time-delay systems have been done. By applying a delay-partitioning approach, the delay-dependent H_∞ analysis and control synthesis for singular systems with constant time delay are studied in [17]. In [18], the robust H_∞ performance analysis for uncertain discrete-time singular systems with time-varying delays is addressed. The state feedback robust H_∞ control problem of discrete-time singular systems with norm-bounded uncertainties and interval time-varying delays in state and input is investigated in [19].

Some works have been done on the research of dissipativity theory. In [6], the issues of strictly dissipative control are studied for discrete singular systems with and without norm-bounded uncertainties, while time delay is not considered. By using the delay partitioning technique, delay-dependent α-dissipativity analysis of continuous time singular systems with constant time delay is investigated in [4]. The problem of robust reliable dissipative filtering for uncertain discrete-time singular system with interval time-varying delay and sensor failures is concerned with

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in [7]. The dissipative control problem for continuous-time linear Markovian jump systems with time-varying delays is addressed with extended dissipativity analysis in [20]. By utilizing the theory of extended dissipativity, the authors in [21] carry out a unified system analysis for neural networks with time-varying delays. The sampled-data extended dissipative control of uncertain Markov jump systems is investigated in [22] by using an input delay approach. In [23], through adopting a novel extended dissipation inequality and a parameter-dependent Lyapunov function, a criterion is obtained to guarantee the extended dissipativity of synchronization error system for chaotic neural networks. In [24], the delay-dependent problems of dissipative analysis and state-feedback synthesis of singular time-delay systems with polytopic uncertainties are studied. The dissipativity analysis for continuous singular systems with time-varying delays is considered in [25]. However, to our best knowledge, there are few works done on the dissipativity analysis for the discrete-time singular system with time-varying delay which motivates us to do this research.

In this paper, strict $(\mathcal{Q},\mathcal{S},\mathcal{R})$-$\alpha$-dissipativity analysis of the discrete singular system with time-varying delay is considered. We first put forward a new time-delay bounded method by using discrete Wirtinger-based inequality proposed in [26] which is less conservative than Jensen inequality. To our best knowledge, this is the first time to apply the discrete Wirtinger-based inequality to study the dissipativity of the discrete singular system. Our approach is also combined with the improved reciprocally convex combination inequality proposed in [27] in order to reduce the conservatism. Then we analyze the $(\mathcal{Q},\mathcal{S},\mathcal{R})$-$\alpha$-dissipativity of the considered systems, which generalizes the H_∞ and passivity performance in a unified framework. Finally, numerical examples are illustrated to show the effectiveness of the proposed method.

Notation: Throughout this paper, $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ represent the n-dimensional Euclidean space and the set of all $n\times m$ real matrices, respectively. The set $\mathbb{S}_n^+$ refers to the set of symmetric positive definite matrices. For symmetric matrix X, $X>0$ (≥ 0) means that X is a positive definite (semi-definite) matrix. I denotes the identity matrix and 0 is a zero matrix with compatible dimensions. The superscript T represents the transpose of the matrices, while $\star$ represents the symmetric terms in a symmetric matrix. sym(A) is defined as $A+A^T$.

2. Problem formulation

Consider discrete-time singular systems with time-varying delay described by

$$\begin{aligned} &Ex(k+1)=Ax(k)+A_d x(k-d(k))+B_\omega \omega(k)\\ &z(k)=Lx(k)+L_d x(k-d(k))+G_\omega \omega(k)\\ &x(k)=\phi(k), k\in[-d_2,0] \end{aligned} \tag{1}$$

where $x(k)\in\mathbb{R}^n$ is the state vector; $\omega(k)\in\mathbb{R}^l$ represents a set of exogenous inputs which includes disturbances to be rejected; $z(k)\in\mathbb{R}^q$ is the control output; $d(k)$ is a time-varying delay satisfying $0<d_1\leq d(k)\leq d_2$, where d_1 and d_2 are prescribed positive integers representing the lower and upper bounds of the time delay, respectively. $\phi(k)$ is the compatible initial condition. The matrix $E\in\mathbb{R}^{n\times n}$ may be singular, and it is assumed that rank $(E)=r\leq n$. A, A_d, B_ω, L, L_d and G_ω are known real constant matrices with appropriate dimensions.

The following lemmas and definitions presented will be used in the proof of the primary results in this paper.

Denote $y(k)=x(k+1)-x(k)$ and a new inequality is derived in the following lemma.

Lemma 1 (*Nam et al. [26]*). *For a given positive definite matrix R and three given non-negative integers a, b, k satisfying $a\leq b\leq k$, denote*

$$\chi(k,a,b)=\begin{cases}\dfrac{1}{b-a}\left[2\sum_{s=k-b}^{k-a-1}x(s)+x(k-a)+x(k-b)\right], & a<b\\ 2x(k-a), & a=b\end{cases}$$

Then, we have

$$-(b-a)\sum_{s=k-b}^{k-a-1}y^T(s)Ry(s)\leq-\begin{bmatrix}\Theta_0\\ \Theta_1\end{bmatrix}^T\begin{bmatrix}R & 0\\ 0 & 3R\end{bmatrix}\begin{bmatrix}\Theta_0\\ \Theta_1\end{bmatrix} \tag{2}$$

where

$$\begin{aligned}&\Theta_0=x(k-a)-x(k-b)\\ &\Theta_1=x(k-a)+x(k-b)-\chi(k,a,b)\end{aligned}$$

Remark 1. There is a difference between Lemma 3 in [26] and Lemma 1 in this paper. After checking, when $a<b$, it is found that the sign of $x(k-b)$ in $\chi(k,a,b)$ should be '+' instead of '−' in [26]. Moreover, it is pointed out that there are minor typographical errors in the Lyapunov function. V_3 is written as

$$\begin{aligned}V_3=&\tau_m\sum_{s=-\tau_m}^{-1}\sum_{\nu=k+s}^{k-1}y^T(\nu)S_1y(\nu)+(\tau_a-\tau_m)\sum_{s=-\tau_a}^{\tau_m-1}\sum_{\nu=k+s}^{k-1}y^T(\nu)S_2y(\nu)\\ &+(\tau_M-\tau_a)\sum_{s=-\tau_M}^{\tau_a-1}\sum_{\nu=k+s}^{k-1}y^T(\nu)S_3y(\nu).\end{aligned}$$

However, in order to get the forward difference of V_3 in (25) of [26], it should be corrected as

$$\begin{aligned}V_3=&\tau_m\sum_{s=-\tau_m}^{-1}\sum_{\nu=k+s}^{k-1}y^T(\nu)S_1y(\nu)+(\tau_a-\tau_m)\sum_{s=-\tau_a+1}^{-\tau_m}\sum_{\nu=k+s-1}^{k-1}y^T(\nu)S_2y(\nu)\\ &+(\tau_M-\tau_a)\sum_{s=-\tau_M+1}^{-\tau_a}\sum_{\nu=k+s-1}^{k-1}y^T(\nu)S_3y(\nu).\end{aligned}$$

Lemma 2 (*Park et al. [28]*). *Let n, m be two positive integers and two matrices R_1 in $\mathbb{S}_n^+$ and R_2 in $\mathbb{S}_m^+$. The improved reciprocally convex combination guarantees that if there exists a matrix X in $\mathbb{R}^{n\times m}$ such that $\left[\begin{smallmatrix}R_1 & X\\ X^T & R_2\end{smallmatrix}\right]\geq 0$, then the following inequality holds for any scalar α in the interval (0,1):*

$$\begin{bmatrix}\frac{1}{\alpha}R_1 & 0\\ 0 & \frac{1}{1-\alpha}R_2\end{bmatrix}\geq\begin{bmatrix}R_1 & X\\ X^T & R_2\end{bmatrix} \tag{3}$$

The nominal discrete singular system with time-varying delay of system (1) can be written as

$$\begin{aligned}&Ex(k+1)=Ax(k)+A_dx(k-d(k))\\ &x(k)=\phi(k), k\in[-d_2,0]\end{aligned} \tag{4}$$

Throughout the paper, the following definitions will be adopted.

Definition 1 (*Wo et al. [29]*).

(i) *The pair (E,A) is said to be regular if* det $(zE-A)$ *is not identically zero.*

(ii) *The pair (E,A) is said to be causal if* deg(det $(zE-A)$) = rank (E).

Definition 2 (*Mei [30]*).

(i) The nominal discrete singular system (4) is said to be regular and causal if the pair (E,A) is regular and causal.

(ii) The nominal discrete singular system (4) is said to be stable if for any $\varepsilon>0$, there exists a scalar $\sigma(\varepsilon)>0$ such that for any compatible initial function $\sup_{-\bar{d}\leq k\leq 0}\|\phi(k)\|\leq\sigma(\varepsilon)$, the solution $x(k)$ of system (4) satisfies $\|x(k)\|\leq\varepsilon$ for $k\geq 0$. Furthermore, $x(k)\to 0$ as $k\to\infty$.