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Research Article

Finite-time stabilization for a class of nonholonomic feedforward systems subject to inputs saturation

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ABSTRACT

This paper studies the problem of finite-time stabilization by state feedback for a class of uncertain nonholonomic systems in feedforward-like form subject to inputs saturation. Under the weaker homogeneous condition on systems growth, a saturated finite-time control scheme is developed by exploiting the adding a power integrator method, the homogeneous domination approach and the nested saturation technique. Together with a novel switching control strategy, the designed saturated controller guarantees that the states of closed-loop system are regulated to zero in a finite time without violation of the constraint. As an application of the proposed theoretical results, the problem of saturated finite-time control for vertical wheel on rotating table is solved. Simulation results are given to demonstrate the effectiveness of the proposed method.

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1. Introduction

In this paper, we consider a class of nonholonomic systems described by

$$
\begin{aligned}
\dot{x}_0 &= u_0+\phi_0(t,x_0)\\
\dot{x}_1 &= x_2u_0+\phi_1(t,x_2,\dots,x_n,u_0,u_1)\\
\dot{x}_2 &= x_3u_0+\phi_2(t,x_3,\dots,x_n,u_0,u_1)\\
&\vdots\\
\dot{x}_{n-1} &= x_nu_0+\phi_{n-1}(t,x_n,u_0,u_1)\\
\dot{x}_n &= u_1+\phi_n(t,u_0,u_1)
\end{aligned}
\tag{1}
$$

where $(x_0,x)^T=(x_0,x_1,\dots,x_n)^T\in R^{n+1}$, $u=(u_0,u_1)^T\in R^2$ are the system state and control input, respectively. $\phi_0:R\times R\to R$ and $\phi_i:R\times R^{n-i}\times R\times R\to R$, $i=1,\dots,n$ are C^1 functions and vanish at the origin. Note that the x-subsystem of system (1) has a feedforward-like structure. This implies system (1) is a nonholonomic system in feedforward-like form, which is also called as nonholonomic feedforward system in this paper.

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The control of nonholonomic systems has attracted a great deal of attention in the past decades because it can be used to model many practical systems, such as wheeled mobile robots, car-like vehicle, under-actuated satellites and so on. However, due to the limitation imposed by Brockett's necessary condition [1], this class of nonlinear systems cannot be stabilized by smooth (or even continuous) time-invariant state feedback. As a consequence, the well-developed smooth nonlinear control theory and methodology cannot be directly used. To circumvent this difficulty, with the effort of many researchers a good number of intelligent approaches have been proposed, which can mainly be classified into discontinuous time-invariant feedback [2,3], smooth time-varying feedback [4–6] and hybrid feedback [7]. The interested reader is referred to the early survey paper [8] and recent ones [9,10] for more details. By means of these valid methods, the robust issue of nonholonomic systems has been systematically studied and fruitful results have been obtained over the last years, for example, one can see [11–20] and the references therein. However, the effect of the inputs constraint is omitted in the above-mentioned results.

As a matter of fact, any actuator always has a limitation of the physical inputs and its existence often severely limits system performance, giving rise to undesirable inaccuracy or leading to instability [21,22]. Thus, it is of great significance to study the problem of stabilizing nonholonomic systems subject to inputs saturation. In this regard, some interesting results have also been reported in the literature. For example, the saturated feedback controllers were constructed in [23–26] for nonholonomic wheeled mobile robots in different types. For nonholonomic systems in standard chained driftless form, i.e, system (1) with $\phi_i(\cdot)=0$, the saturated asymptotic stabilizers were constructed in [27,28]. Taking into account that the finite-time stable systems may demonstrate not only faster convergence rates, but also higher accuracies and better disturbance rejection properties [29], Wu et al. [30] recently investigated the saturated finite-time stabilization of system (1) with $\phi_n(\cdot)=0$ under the assumption that $|\phi_0(\cdot)| \le M$ and in a neighborhood of the origin, the following holds:

$$|\phi_i(\cdot)| \le b\sum_{j=i+1}^{n} |x_j|^{q_{ij}}, \quad i=1,\dots,n-1 \tag{2}$$

where constants $M \ge 0$, $b>0$, $-1/n<\tau<0$ and q_{ij} satisfy $q_{ij} > (1+i\tau)/(1+(j-1)\tau) \ge 1$. Note that both the upper bound of ϕ_i's independent of inputs and $q_{i,i+1}>1$ are required in this assumption. However, there exist practical systems whose nonlinear terms do not satisfy such restriction, such as the vertical wheel on rotating table presented in Section 4. As a natural extension, the following interesting problem is proposed: *Is it possible to further relax this nonlinear growth condition? Under the weaker condition, can a saturated finite-time stabilizing controller be designed ?*

Motivated by the above observation, we shall address this problem here and provide a solution to the problem of finite-time stabilization of nonholonomic feedforward system (1) by using saturated state feedback. The contributions of this paper are three-folds: (i) By comparison with the existing results in [30–40], the nonlinear growth condition is largely relaxed and a much weaker sufficient condition is given. (ii) Based on a combined application of the adding a power integrator method, the homogeneous domination approach and the nested saturation technique, a new systematic saturated state feedback control design procedure is proposed to solve the finite-time stabilization problem for all plants in the considered class and leads to more general results never achieved before. (iii) An application example for vertical wheel on rotating table is modeled and solved by the proposed method.

Notations: Throughout this paper, the following notations are adopted. R^+ denotes the set of all nonnegative real numbers and R^n denotes the real n-dimensional space. For a given vector or matrix X, X^T denotes its transpose, and $|X|$ is the Euclidean norm of a vector X. C^i denotes the set of all functions with continuous ith partial derivatives. A sign function sign(x) is defined as follows: sign$(x)=1$, if $x>0$; sign$(x)=0$, if $x=0$ and sign$(x)=-1$, if $x<0$. For any $a \in R^+$ and $x \in R$, the function $[x]^a$ is defined as $[x]^a = \text{sgn}(x)|x|^a$. Besides, let $\sum_j^i(\cdot)=0$ if $j>i$ and the arguments of the functions will be omitted or simplified, whenever no confusion can arise from the context. For instance, we sometimes denote a function $f(x(t))$ by simply $f(x)$, $f(\cdot)$ or f.

2. Problem formulation and preliminaries

The objective of this paper is to present a saturated control design strategy which stabilizes the system (1) in a finite time under the following saturation constraint:

$$-u_i^{max} \le u_i \le u_i^{max}, \quad i=0,1 \tag{3}$$

where $u_0{}^{max}$ and $u_1{}^{max}$ are priori known positive real numbers.

To this end, the following assumptions are imposed in this paper.

Assumption 2.1. There is a constant $M \ge 0$ such that

$$|\phi_0(\cdot)| \le M < u_0^{max} \tag{4}$$

Assumption 2.2. For $i=1,\dots,n$, there are constants $0 \le a < \min\{1, u_0^{max}\}$, $b>0$ and $\tau \in (-1/n, 0)$ such that

$$|\phi_i(\cdot)| \le a|x_{i+1}| + b\sum_{j=i+2}^{n+1} |x_j|^{(r_i+\tau)/r_j} \tag{5}$$

where $x_{n+1}=u_1$ and $r_i = 1+(i-1)\tau$, $i=1,\dots,n+1$.

Remark 2.1. Although Assumption 2.1 seems to be somewhat restrictive, it is very necessary to ensure the existence of saturated stabilizer for nonholonomic system (1). Next, we will explain its necessity from two points.

(i) *The boundedness of ϕ_0 is necessary*. If not, the x_0-subsystem might be uncontrollable. For example, consider the following simple case $\phi_0 = x_0^2$. Obviously, when $|u_0| \le u_0^{max}$ is needed, there does not exist any saturated control to globally stabilize this system with initial value $x_0(0) > u_0^{max}+1$.

(ii) *$M < u_0^{max}$ is necessary*. Similarly, when $\phi_0 \ge d_n \ge u_0^{max}$, the solution of x_0-subsystem satisfies $|x_0(t)| \ge |x_0(0)|e^{(d_0-u_0^{max})t}$, which leads to that there does not exist any saturated control to globally stabilize such system.

Remark 2.2. It is worth pointing out that most of the existing results on nonlinear feedforward systems require the upper bound of nonlinear term ϕ_i being independent of state x_{i+1} or input u_1, for example, the x_{i+1}-free-growth is needed in [31–36] and the u_1-free-growth is needed in [30,37–40], Assumption 2.2, in which both state x_{i+1} and input u_1 beside the states $x_{i+2},\dots,x_n$ are involved, is less restrictive and allows for a much broader class of systems in some sense.

Remark 2.3. The boundedness and C^1 property of ϕ_0 in Assumption 2.1 imply that there exist constants $c>0$ and $\omega \in (0,1)$ such that $|\phi_0(\cdot)| \le c|x_0|^{\omega}$.

In what follows, we review some useful definitions and lemmas which will serve as the basis of the coming control design and performance analysis.

Definition 2.1 (*Wu et al. [20]*). Consider the nonlinear system

$$\dot{x} = f(t,x) \quad \text{with } f(t,0)=0, \quad x \in R^n \tag{6}$$

where $f: R^+ \times U_0 \to R^n$ is continuous with respect to x on an open neighborhood U_0 of the origin $x=0$. The equilibrium $x=0$ of the system is (locally) uniformly finite-time stable if it is uniformly Lyapunov stable and finite-time convergent in a neighborhood $U \subseteq U_0$ of the origin. By "finite-time convergence," we mean: If, for any initial condition $x(t_0) \in U$ at any given initial time $t_0 \ge 0$, there is a settling time $T>0$, such that every $x(t,t_0,x(t_0))$ of system (6) is defined with $x(t,t_0,x(t_0)) \in U/\{0\}$ for $t \in [t_0,T)$ and satisfies $\lim_{t\to T} x(t,t_0,x(t_0))=0$ and $x(t,t_0,x(t_0))=0$ for any $t \ge T$. If $U=U_0=R^n$, the origin is a globally (uniformly) finite-time stable equilibrium.

Lemma 2.1 (*Wu et al. [20]*). *Consider the nonlinear system described in (6). Suppose there is a C^1 function $V(t,x)$ defined on $\hat{U} \subseteq U_0 \times R$, where $\hat{U}$ is a neighborhood of the origin, class K functions π_1 and π_2, real numbers $c>0$ and $0<\alpha<1$, for $t \in [t_0,T)$ and $x \in \hat{U}$ such that (i) $\pi_1(|x|) \le V(t,x) \le \pi_2(|x|)$, $\forall t \ge t_0$, $\forall x \in \hat{U}$; (ii) $\dot{V}(t,x)+cV^{\alpha}(t,x) \le 0$, $\forall t \ge t_0$, $\forall x \in \hat{U}$. Then, the origin of (6) is uniformly finite-time stable with $T \le \frac{V^{1-\alpha}(t_0,x(t_0))}{c(1-\alpha)}$ for initial condition $x(t_0)$ in some open neighborhood $\hat{U}$ of the origin at initial time t_0. If $\hat{U}$*

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